Vertical Exclusion with Downstream Risk Aversion or Limited Liability*

Stephen Hansen
University of Oxford
Alan Turing Institute
stephen.hansen@economics.ox.ac.uk

Massimo Motta
ICREA-UPF
Barcelona GSE
massimo.motta@upf.edu

December 22, 2018

Abstract

An upstream firm with full commitment bilaterally contracts with two ex ante identical downstream firms. Each observes its own cost shock, and faces uncertainty from its competitor's shock. When they are risk neutral and can absorb losses, the upstream firm contracts symmetric outputs for production efficiency. However, when they are risk averse, competition requires the payment of a risk premium due to revenue uncertainty. Moreover, when they enjoy limited liability, competition requires the upstream firm to share additional surplus. To resolve these trade-offs, the upstream firm offers exclusive contracts in many cases.

Keywords: Exclusive Contracts, Risk, Limited Liability

JEL Codes: L22, L42, D86

*The authors gratefully acknowledge financial aid from the Spanish Ministry of Science and Innovation (projects ECO2010-15052 and ECO2013-43011-P). We would like to thank Larbi Alaoui for extensive comments on previous versions of this paper, as well as Mark Armstrong, Giacomo Calzolari, Yutian Chen, Simon Cowan, Jan Eeckhout, Péter Eső, Ricard Gil, Liliane Karlinger, Clare Leaver, Marco Ottaviani, Patrick Rey, Monika Schnitzer, Emanuele Tarantino, John Thanassoulis, participants of the ANR-DFG workshop (Paris), and seminar attendees at Bologna, Carlos III, Mannheim, and Oxford for helpful advice. Finally, we are grateful to the Editor and two referees for their comments. Previous versions of this paper had the title "Vertical Exclusion with Endogenous Competition Externalities."

1 Introduction

Very often, a manufacturer has to decide whether to sell its products through one or several retailers, a franchisor whether to have one or multiple franchisees, the owner of a patent whether to license its technology to one or more licensees. In many cases, a situation arises in which only one agent will deal with the principal's product, brand, or technology.[1] This paper provides a novel rationale for the optimality of such exclusive relationships and, more generally, asymmetric market shares at the retail level. We argue that competition with imperfectly correlated and privately observed cost or demand shocks necessarily creates uncertainty for downstream retailers, and that exclusive contracts can therefore benefit upstream firms when these retailers are subject to risk aversion or limited liability.

Our analysis builds on two basic ideas. First, when downstream firms compete, they impose externalities on one another. If one produces more, the market price goes down, affecting the profits of others. But if production levels depend on the realization of individual shocks that are not fully observable to competitors, downstream firms do not know the size of the externality that competition will impose on them. Second, the size of the externalities is endogenous to the contracts the upstream firm offers. By increasing or decreasing the difference in input levels offered to firms, the upstream firm determines the magnitude of the uncertainty that downstream firms face. In short, much of the literature focuses on the effect of competition on average downstream profits, or the first moment of the payoff distribution. We instead study the effect of competition on the second moment, and the resulting implications for market structure.

When downstream firms are risk neutral and can absorb losses, the upstream firm has an inherent incentive to offer all of them the input because doing so ensures that whenever a more productive firm exists it serves some of the market.[2] When downstream firms are risk averse, there is a cost to the upstream firm of contracting more than one: the uncertainty in their realized profit forces it to pay them a risk premium. When risk aversion is sufficiently high, exclusive contracts are optimal because, by offering zero input to all but one downstream firm, the upstream firm eliminates the competition externality, and with it the uncertainty from competition that downstream firms face.[3] A similar mechanism operates when downstream firms enjoy limited liability. The resulting

[1]According to Lafontaine and Slade (2008), one third of retail sales through independent outlets occur in exclusive relationships. Further evidence on this point comes from Blair and Lafontaine (2011), who analyze a large dataset of franchise contracts, and show that, in 17 out of 18 sectors, more than 50% of franchisors adopt exclusive territories. In the context of licensing deals, Anand and Khanna (2000) show that over 30% of the agreements in their dataset are exclusive.

[2]One can think of other mechanisms, such as product differentiation, that would also generate the optimality of offering more than one firm the input.

[3]We also show that, for intermediate risk preferences and two firms, partial exclusion arises, with one downstream firm producing more for all shock realizations.

surplus that the upstream firm must leave when contracting with two firms more than offsets the gain from production efficiency, which induces it to offer an exclusive contract under certain conditions.

This paper builds on a long literature on bilateral contracting[4] in vertical markets (Hart and Tirole 1990, McAfee and Schwartz 1994, Segal 1999, Rey and Tirole 2007) that largely focuses on exclusion as a response to commitment problems. When the upstream firm can commit to public bilateral contracts, it can always extract the monopoly profit; but when it offers unobservable bilateral contracts, it cannot commit not to renegotiate with downstream firms and does not obtain the monopoly profit. Exclusive contracts serve as a commitment device to restore monopoly profit. Our mechanism is wholly different from this one. In the model the upstream firm has full commitment, but chooses exclusive outcomes to reduce uncertainty in the downstream market.

The most closely related paper to ours in the literature is Rey and Tirole (1986), who also study a vertical market with bilateral contracting in which downstream firms are subject to shocks. In their model, when downstream firms are infinitely risk averse the upstream firm allows them to compete, while with risk neutrality it offers exclusive territories. The key difference with our paper is that in Rey and Tirole (1986) downstream shocks are perfectly correlated so that market structure does not affect downstream uncertainty. We instead show that competition creates uncertainty when shocks are i.i.d. (as in the model of section 3) or more generally imperfectly correlated (see extension in section 4.1), and as a result arrive at the exact opposite conclusion to that of Rey and Tirole (1986). In our model, when downstream firms are risk neutral the upstream firm allows competition, but when they are infinitely risk averse it provides a fully exclusive contract. We are not aware of another paper in the literature that focuses on exclusion as a response to the uncertainty that market competition creates.

In an environment similar to ours, Dequiedt and Martimort (2015) study bilateral contracts in which the upstream firm can adjust the contract offered to any single downstream firm based on what it learns about the costs of other firms during the contracting process. In the optimal contract, all downstream firms pay a cost-dependent fixed fee, and the entire market is allocated to the lowest-cost firm, which can be interpreted as an exclusive outcome.[5] The fundamental force leading to exclusion is screening. In our

[4]A bilateral contract between an upstream and downstream firm cannot directly depend on the outputs or messages of other firms. Motivations for this restriction include the transaction costs associated with writing and enforcing multilateral contracts, and the possibility that multilateral contracts might facilitate collusion. Although vertical contracts are typically regarded as having less anticompetitive potential than horizontal contracts, antitrust authorities are often concerned with *contracts that reference rivals*, that is, vertical contracts between a buyer and a seller whose terms may depend on information or contract terms pertaining to the buyer's rivals (as in the situation depicted in our paper) or the seller's rivals (Scott Morton 2012).

[5]This outcome is similar to that with multilateral contracting. See McAfee and McMillan (1986), Laffont and Tirole (1987), McAfee and McMillan (1987), Riordan and Sappington (1987), and Dasgupta

model, instead, the upstream firm simply posts contracts for each downstream firm and does not adjust them depending on information it may gather from other downstream firms. The fundamental force leading to exclusion is the elimination of a surplus that must be paid to competing downstream firms that are risk averse or protected by limited liability. In both models, the outputs and transfers paid by a downstream firm depend on its own costs; the key difference is whether these vary in *other* firms' costs as well. In practice, it appears that many upstream firms do not adjust terms-of-trade for individual firms as a function of the cost structure of all potential retailers. Lafontaine and Shaw (1999) show in a large panel dataset of franchisors that 75% offer identical contracts to all downstream firms in their network during a thirteen-year period. Lafontaine (1992) surveys franchisors about their contractual process, and 42% report offering contracts on a take-it-or-leave it basis with no possibility for negotiation, while another 38% allow no negotiation over monetary terms. Our paper describes well situations in which the upstream firm posts contracts for downstream firms that do not depend on communication with them. Obviously this assumption does not fit every vertical market, but we find it empirically plausible in many.

Finally, while limited liability is a rather standard assumption in the industrial-organization theory literature, risk aversion is not. However, the empirical literature has long recognized its importance. For example, the majority of exclusive retailing occurs in franchise networks (Lafontaine and Slade 2008), within which the owners of retail outlets are typically small and undiversified; some franchisors even explicitly seek out retailers whose incomes are highly correlated with their outlets' performance (Kaufmann and Lafontaine 1994). Asplund (2002) and Banal-Estañol and Ottaviani (2006) also present numerous references to empirical studies documenting the relevance of firm risk aversion.[6] More generally, Nocke and Thanassoulis (2014) provide theoretical foundations for introducing curvature into downstream firms' payoff functions. They show that when downstream firms face credit constraints subsequent to competing in the downstream market, they behave as if they were risk averse even if they are risk neutral.

The paper is organized thus. Section 2 describes the model and presents a solution of the baseline case without risk aversion or limited liability. Section 3 then considers the impact of risk aversion and limited liability on exclusive arrangements. Section 4 contains a few extensions of the model. Finally, section 5 provides a discussion and concludes. *Appendix A contains all proofs.*

and Spulber (1989).

[6]They also make a related point to ours in horizontal markets by observing that mergers have a role to play in reducing the uncertainty that firms face.

2 Model

Consider a vertical market in which an upstream firm supplies an input that is transformed into output in a one-to-one relationship by two downstream firms $i = 1, 2$. Aggregate demand for the product is $P(Q)$ where $Q \geq 0$ is aggregate quantity. We assume that $P'(Q) < 0$, and that marginal revenue $MR(Q) \equiv P(Q) + QP'(Q)$ is decreasing. Finally, for technical reasons, we also assume that the upstream firm faces a capacity constraint such that it cannot supply more than $\overline{Q}$ units of input, where $\overline{Q}$ is an arbitrarily large but finite quantity.

Each downstream firm has a constant marginal cost of production $c_i \in \{0, c\}$ where $c > 0$. Each firm observes the realization of its own cost shock, but not that of its competitor. The upstream firm observes neither shock. Shocks are iid with $\Pr[c_i = 0] = r$. Constant returns to scale keep aggregate production costs independent of the distribution of output across downstream firms, so that we can isolate the impact of revenue uncertainty. The interpretation of c_i as a cost shock is simply for concreteness, as it can equally represent a demand shock.[7] Finally, we assume that $c < P\left(MR^{-1}(0)\right)$.[8]

The upstream firm offers the nonlinear wholesale price contract $T_i(Q_i)$ to downstream firm i with the interpretation that i commits to pay $T_i(Q_i)$ to the upstream firm when producing Q_i units of output. Given these contracts, which are publicly observed, firms engage in Cournot competition and simultaneously choose outputs. Profits are then realized and payments are made to the upstream firm. Downstream firms can guarantee zero profits by exiting the market without producing.

By the revelation principle, we can focus on the upstream firm's offering downstream firm i an incentive-compatible two-point contract $[Q_i(\hat{c}_i), T_i(\hat{c}_i)]$ for $\hat{c}_i \in \{0, c\}$ in which i prefers to truthfully report its realized marginal cost. Let

$$\pi_i(\widehat{c}_i, \widehat{c}_j, c_i) = Q_i(\widehat{c}_i)P\left[Q_i(\widehat{c}_i) + Q_j(\widehat{c}_j)\right] - Q_i(\widehat{c}_i)c_i - T_i(\widehat{c}_i) \tag{1}$$

be firm i's profit from reporting cost type $\widehat{c}_i$ when the competitor reports cost type $\widehat{c}_j$ and firm i has marginal cost c_i. A key feature of the model is the uncertainty regarding the competitor's cost shock which in turn generates revenue uncertainty.

In incentive-compatible contracts, firm i faces the lottery

$$L_i(\widehat{c}_i \mid c_i) = \left\{\left[\pi_i(\widehat{c}_i, 0, c_i), \pi_i(\widehat{c}_i, c, c_i)\right]; (r, 1-r)\right\} \tag{2}$$

[7]Consider a differentiated products model in which firm i's demand is $p_i = v_i - Q_i - \gamma \sum_{j \neq i} Q_j$ and its profit is $\pi = (p_i - c_i)Q_i$. For $\gamma \to 1$, whether the shock is on v_i or c_i is formally equivalent.

[8]This is a standard condition. It implies that low cost firms face competition from high cost firms in the sense that the high cost firm can still profitably produce when the low cost firm chooses its monopoly quantity, which is precisely $MR^{-1}(0)$.

when reporting cost type $\widehat{c}_i$. We assume firms use the constant absolute risk aversion (CARA) utility function $u(\pi_i) = -\exp(-a\pi_i)$ to evaluate the expected utility of the lottery, which we denote $U[L_i(\widehat{c}_i \mid c_i)]$. The a parameter is the coefficient of absolute risk aversion, and higher values indicate more risk aversion. a is common knowledge and shared by both downstream firms.

The upstream firm's problem can be written as

$$\max_{\{Q_i(c_i),T_i(c_i)\}_{i=1,2;c_i\in\{0,c\}}} \sum_{i=1}^{2} rT_i(0) + (1-r)T_i(c) \text{ such that} \tag{3}$$

$$U[L_i(c_i \mid 0)] \geq 0 \tag{PC}$$

$$U[L_i(c_i \mid c_i)] \geq U[L_i(c_j \mid c_i)] \text{ for } c_j \neq c_i \tag{IC}$$

$$\overline{Q} \geq Q_i(c_i) \geq 0. \tag{QQ}$$

Since firms can earn zero profit from exiting the market, their contracts must provide them at least 0 to ensure participation. The IC constraints ensure firms report their realized cost shock truthfully, while the QQ constraints express the required bounds on output.

Both downstream firms are symmetric, in the sense that they have the same distribution over cost shocks and the same utility function over lotteries. We are interested in situations in which the upstream firm nevertheless induces asymmetric outcomes in the downstream market due to revenue uncertainty. There are two relevant definitions of exclusion.

Definition 1 *Let* $\{Q_i^*(c_i), T_i^*(c_i)\}_{i=1,2;c_i\in\{0,c\}}$ *be a solution to (3).*

1. *Contracts are uniform if* $Q_1^*(c_1) = Q_2^*(c_2)$ *whenever* $c_1 = c_2$.
2. *Firm* i *is partially excluded if* $0 < Q_i^*(c_i) < Q_j^*(c_j)$ *for* $j \neq i$ *whenever* $c_i = c_j$.
3. *Firm* i *is fully excluded if* $Q_i^*(0) = Q_i^*(c) = 0$.

It is important to emphasize that we define exclusion as an equilibrium outcome of the Cournot game played between downstream firms rather than as an explicit contractual clause: the upstream firm simply posts contracts and lets downstream firms choose outputs given the terms of the contracts. Moreover, the direct mechanisms we analyze are not the only contracts that implement exclusive outcomes. For example, in the analysis we show conditions under which the upstream firm can implement fully exclusive outcomes while offering both downstream firms the same contract. As we discuss below, the fact that exclusion is implicit rather than explicit presents challenges for regulators.

2.1 Simplified program

In lottery (2), the uncertainty comes from the revenue side, whereas the production cost $Q_i(\widehat{c}_i)c_i$ and transfer $T_i(\widehat{c}_i)$ are deterministic. CARA utility allows one to linearly separate these and represent expected utility as

$$U\left[L_i(\widehat{c}_i \mid c_i)\right] = \text{CertRev}_i(\widehat{c}_i) - Q_i(\widehat{c}_i)c_i - T_i(\widehat{c}_i).$$

Here $\text{CertRev}_i(\widehat{c}_i)$ is certainty-equivalent revenue, or the fixed payment net of production and transfer costs that gives the firm the same expected payoff as (2).

This representation of expected utility allows one to write program (3) in a simpler way using standard arguments from the mechanism design literature. The basic idea is to solve the principal's program considering only the participation constraints of the high-cost firms and incentive-compatibility constraints of the low-cost firms (see the proof of 1 in the Appendix for details).

Lemma 1 *Program* (3) *is equivalent to maximizing*

$$\sum_i \left[r\,\text{CertRev}_i(0) + (1-r)\,\text{CertRev}_i(c)\right] - \sum_i cQ_i(c) \tag{4}$$

such that $\overline{Q} \geq Q_i(0) \geq Q_i(c) \geq 0$.

The representation of the upstream firm's expected profits in lemma 1 has an intuitive form. The first summation is expected certainty-equivalent revenue. The second summation represents two kinds of costs. The first is the expected production costs $\sum_i (1-r)cQ_i(c)$ that are only incurred by high-cost downstream firms. The second is the information rent that must be left to low-cost downstream firms, which in expected value terms is $\sum_i rcQ_i(c)$. Summing both costs gives $\sum_i cQ_i(c)$. As for the constraints, $Q_i(0) \geq Q_i(c)$ is a necessary condition for incentive compatibility, and means that efficient firms produce more than inefficient ones in equilibrium.

In deriving optimal contracts, it is useful to define $Q_i^H \equiv Q_i(c)$, $\Delta_i \equiv Q_i(0) - Q_i^H$, $Q^H \equiv Q_1^H + Q_2^H$, and $\Delta \equiv \Delta_1 + \Delta_2$. Q^H and Δ are aggregate production variables, while Q_i^H and Δ_i are distribution variables. When these variables carry asterisk superscripts, they should be understood to represent optimal values.

To complete the preliminaries, we provide conditions under which the upstream firm wishes to contract positive aggregate output from both cost types.

Lemma 2 *For all parameter values* $\Delta^* > 0$. *There exist values of* r^* *and* c^* *such that* $Q^{H*} > 0$ *whenever* $r < r^*$ *and* $c < c^*$.

In other words, the upstream firm always contracts positive output from efficient downstream firms, but only contracts positive output from inefficient firms if they are sufficiently likely to be present (r low) and not too inefficient (c low). Otherwise, the upstream firm wishes to shut out high-cost firms entirely. We assume throughout the paper that $r < r^*$ and $c < c^*$ since the main question of interest is not whether production occurs at all, but how production is distributed across downstream firms given risk aversion. Section 4 discusses the situation with multiple cost types.

2.2 Baseline solution

We begin by analyzing a baseline case in which downstream firms are neither risk averse nor subject to limited liability. Here the only force that affects the upstream firm's choice is production efficiency, which our first result shows leads to the optimality of uniform contracts.

Proposition 1 *When firms are risk neutral and do not face limited liability, uniform contracts are (weakly) optimal. In particular,* $\Delta_1^* = \Delta_2^* = \frac{\Delta^*}{2} > 0$*, while profits are independent of the distribution of* Q^{H*} *across firms.*

The basic intuition for the result is that the upstream firm wants efficient downstream firms to produce equal amounts to avoid a situation in which it must rely mainly on a high-cost producer to serve the market when a low-cost producer is also available. This is seen most clearly in the linear demand case where $P = 1 - Q$ and expected revenue is $\mathbb{E}[\,Q(1-Q)\,] = \mathbb{E}[\,Q\,] - \mathbb{E}[\,Q\,]^2 - V[\,Q\,]$. The upstream firm should therefore distribute output across the two firms to decrease the variance in aggregate output, which one can easily show is $V[\,Q\,] = r(1-r)\sum_i \Delta_i^2$. This is clearly minimized by equating Δ_i across firms. Essentially, having two firms in the market helps the upstream firm "hedge its bets" by making sure that when one of the two firms is the low cost type it gets a piece of the market.

For the more general argument, we first observe that certainty equivalent revenue is simply expected revenue so that

$$\text{CertRev}_i^{RN}(\widehat{c}_i) = rQ_i(\widehat{c}_i)P[Q_i(\widehat{c}_i) + Q_j(0)] + (1-r)Q_i(\widehat{c}_i)P[Q_i(\widehat{c}_i) + Q_j(c)]. \quad (5)$$

Using the notation described at the end of section 2.1, we can write the upstream firm's objective function in (4) as

$$\begin{gathered} r^2\left(Q^H + \Delta\right)P\left(Q^H + \Delta\right) + (1-r)^2 Q^H P\left(Q^H\right) + \\ r(1-r)\left(Q^H + \Delta_1\right)P\left(Q^H + \Delta_1\right) + r(1-r)\left(Q^H + \Delta - \Delta_1\right)P\left(Q^H + \Delta - \Delta_1\right) - \\ cQ^H \end{gathered} \quad (6)$$

The terms on the first two lines of (6) refer to total downstream revenue for different realizations of downstream costs. For example, $(Q^H + \Delta)P(Q^H + \Delta)$ is the total revenue when both firms are low cost, which occurs with probability r^2. Upstream profits depend on the distribution of output only through Δ_1, and this in turn is only relevant when one downstream firm is low cost and the other is high cost (reflected in the second line of (6)). When firm 1 is low cost and firm 2 is high cost, the marginal impact of raising Δ_1 is $\text{MR}\left(Q^H + \Delta_1\right)$, while in the opposite case the marginal impact is $-\text{MR}\left(Q^H + \Delta - \Delta_1\right)$. For a fixed value of Δ, the optimal Δ_1 is therefore defined by

$$\text{MR}\left(Q^H + \Delta_1^*\right) = \text{MR}\left(Q^H + \Delta - \Delta_1^*\right). \tag{7}$$

Proposition 1 pins down the distribution of Δ^* across downstream firms, but not that of Q^{H*}. As long as $\Delta_1^* = \Delta_2^*$ holds, any split of Q^{H*} is optimal: as can be observed from (6) upstream profit is the same when $\Delta_1^* = \Delta_2^* = \frac{\Delta^*}{2}$ and $Q_1^{H*} = Q_2^{H*} = \frac{Q^{H*}}{2}$ as when $\Delta_1^* = \Delta_2^* = \frac{\Delta^*}{2}$, $Q_1^{H*} = \frac{Q^{H*}}{2} + K$, and $Q_2^{H*} = \frac{Q^{H*}}{2} - K$ for some constant $0 < K \leq \frac{Q^{H*}}{2}$. However, there is no fundamental force driving asymmetric outcomes, and we view uniform contracts as the natural ones.

3 Exclusion and Risk Aversion

We now depart from the baseline case and assume that downstream firms are risk averse. If serving two firms is useful for the upstream firm because of production efficiency, it also creates uncertainty for downstream firms. When a downstream firm sells in the market alone, it knows for certain what its profits will be. On the other hand, when facing a competitor whose output level varies with its cost shocks, profits are uncertain. In the case of risk neutrality, this has no effect on downstream firms' utility. In reality, however, one might imagine that downstream firms have some aversion to the uncertainty that competition creates.

3.1 High risk aversion

We begin by analyzing a situation in which downstream risk aversion is high, which corresponds to a large value of a in the utility function. The next results shows the dramatic consequences for optimal production.

Proposition 2 *There exists a finite $\overline{a}$ such that fully exclusive contracts are strictly optimal for all $a > \overline{a}$.*

In words, with sufficiently high risk aversion, the upstream firm supplies just one of the two downstream firms (since they are symmetric, the upstream firm can choose to exclusively deal with either). To proceed with the argument, we first analyze the case of infinite risk aversion (obtained as $a \to \infty$) and then use continuity arguments to extend the result to finite values of a.

In contrast to the situation with risk neutrality, with infinite risk aversion the certainty equivalent revenue becomes

$$\text{CertRev}_i^{IRA}(\widehat{c}_i) = Q_i(\widehat{c}_i)P[Q_i(\widehat{c}_i) + Q_j(0)]. \tag{8}$$

To see why, first note that an infinitely risk averse firm's expected utility from a lottery coincides with its lowest possible realization.[9] Moreover, as established in lemma 1, a necessary condition for incentive compatibility is that low-cost firms produce more than high-cost firms. Since revenue is strictly decreasing in the output of a competitor, the lowest realization of the lottery that each downstream firms faces is the revenue gained from meeting an efficient competitor. Comparing (8) with (5) is instructive because it immediately shows that if the upstream firm offers the optimal symmetric contracts for risk neutral downstream firms to infinitely risk averse downstream firms, then its profits in (4) are strictly lower. In this sense, risk aversion in downstream firms increases the costs of serving both of them. The question is: how does the upstream firm respond to risk aversion in its optimal contract?

Plugging (8) into (4) gives upstream profit with infinite risk aversion as

$$\begin{gathered} r\left(Q^H + \Delta\right) P\left(Q^H + \Delta\right) + \\ (1-r)\left[Q_1^H P\left(Q^H + \Delta - \Delta_1\right) + (Q^H - Q_1^H) P\left(Q^H + \Delta_1\right)\right] - cQ^H. \end{gathered} \tag{9}$$

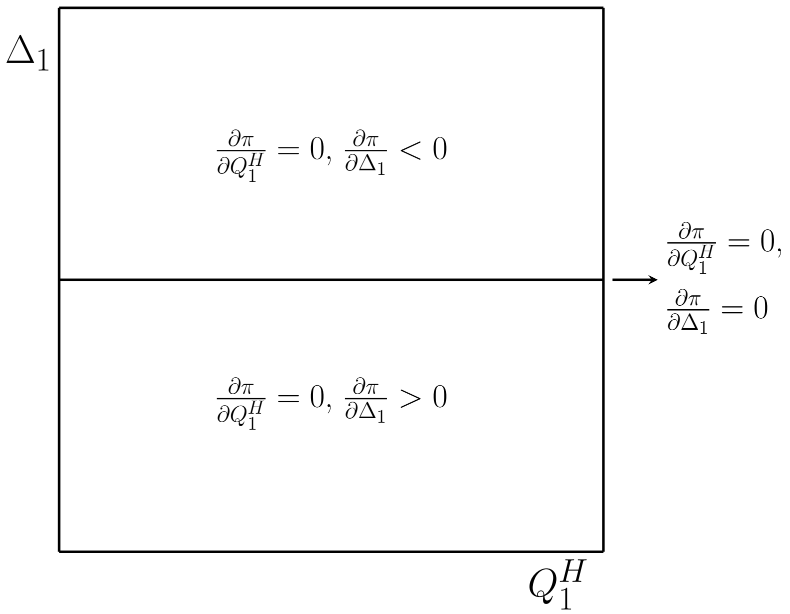
The optimality of fully exclusive contracts is then immediate. Since price is decreasing in output, the term is square brackets in (9) is less than $Q_1^H P(Q^H) + (Q^H - Q_1^H)P(Q^H) = Q^H P(Q^H)$. So (9) has an upper bound of

$$r\left(Q^H + \Delta\right) P\left(Q^H + \Delta\right) + (1-r)Q^H P(Q^H) - cQ^H,$$

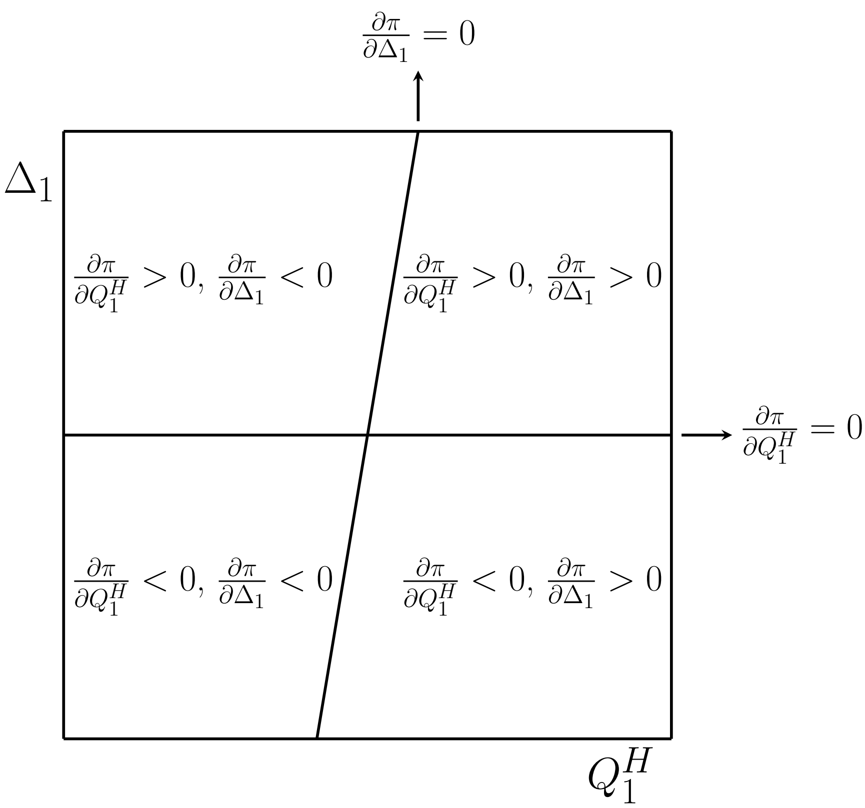
which is precisely the expected profit the upstream firm generates by contracting with a single firm. The upstream firm can only generate profits through the transfer payments that downstream firms are willing to make, and with infinite risk aversion it cannot extract any of the benefit of productive efficiency since downstream firms' risk premium fully swamps the profits they make when facing an inefficient competitor.

Figure 1 provides further intuition about how high risk aversion changes the optimal contract. Figure 1a shows the impact on upstream profits of changes in Q_1^H and Δ_1 with risk neutrality. As discussed in the baseline solution, Q_1^H has no effect while moving Δ_1

[9] By definition, an infinitely risk averse firms' expected utility over a lottery with outcomes $(x_1, \ldots, x_N)$ and associated probabilities $(p_1, \ldots, p_N)$ is $\min\{x_1, \ldots, x_N\}$.



(a) Risk neutrality



(b) Infinite risk aversion

Figure 1: Derivatives of upstream profits π in distribution variables

This figure illustrates the signs of the derivative of the upstream firm's objective function in Q_1^H and Δ_1 for fixed $Q^H > 0$ and $\Delta > 0$ in the cases of risk neutrality and infinite risk aversion, respectively. In each subfigure, Q_1^H is plotted on the horizontal axis, whose length is Q^H and Δ_1 on the vertical, whose length is Δ. The locus of points at which $\frac{\partial \pi}{\partial \Delta_1} = 0$ in the infinite risk aversion case is for illustration only; in general it is not linear.

closer to the even split $\Delta_1 = \frac{\Delta}{2}$ always increases them. In contrast, figure 1b shows the equivalent impact for infinite risk aversion.[10] In this case, the distribution of Q^H across downstream firms does impact upstream profit. To see why, consider a situation in which $\Delta_1 > \Delta_2$. Per unit of production, firm 2 is then facing more risk from competition in the sense that the drop in the market price from meeting an efficient competitor is higher than for firm 1. To mitigate the corresponding drop in profit, the upstream firm can reduce the production of firm 2 by decreasing Q_2^H or, equivalently, by increasing Q_1^H. To see this more formally, the risk premium that must be paid out to low-cost firm 2 is $(1-r)\left\{(Q_2^H + \Delta_2)[P(Q^H + \Delta_2) - P(Q^H + \Delta)]\right\}$, which decreases on the margin with Q_2^H by $(1-r)[P(Q^H+\Delta_2) - P(Q^H+\Delta)]$. The risk premium paid out to firm 1 increases in Q_1^H on the margin by $(1-r)[P(Q^H + \Delta_1) - P(Q^H + \Delta)]$ which is strictly less than $(1-r)[P(Q^H+\Delta_2) - P(Q^H+\Delta)]$. Thus the upstream firm gains by increasing Q_1^H. An equivalent argument can be made for high-cost firms. Moreover, once Q_1^H is sufficiently high, increasing Δ_1 increases upstream profits by shielding firm 1 from the risk of meeting an efficient firm 2.

The results so far already contain a basic message of the paper. Even if the upstream firm can fully commit to bilateral contracts, it may simply be too costly to include both firms in the downstream market due to the uncertainty that competition creates.

3.2 Intermediate risk aversion

We now explore the model for all values of a. In the risk neutral objective function (6) and infinite risk aversion objective function (9), upstream profits are linear in various revenue terms. With CARA utility and $a \in (0, \infty)$, there is instead curvature in revenue in the upstream objective function. To avoid the additional complication of curvature in the demand function, we analyze the linear demand case $P(Q) = 1 - Q$.

The baseline case of section 2.2 showed that the upstream firm chooses symmetric downstream outputs with risk neutrality to enhance production efficiency, while with high risk aversion it chooses starkly asymmetric outcomes to minimize risk. In the intermediate case, the upstream firm responds to both production efficiency and risk premia, but the implications of this are not immediately obvious. For example, for low levels of risk aversion, risk premia are also small, so one might plausibly think that symmetric outcomes remain optimal. Our next result shows that asymmetric outcomes arise for *any* level of risk aversion.

Proposition 3 *When $P = 1-Q$ and $a > 0$, exclusion (partial or full) is strictly optimal.*

To gain an intuition for the result, recall that with risk neutrality the marginal impact on upstream profits of varying Δ_1 around its optimal value $\Delta/2$ is essentially zero since

[10] We formally derive the properties of the derivatives plotted in figure 1 in the appendix.

profits are strictly concave in Δ_1. On the other hand, for $a > 0$, adjusting Δ_1 away from $\Delta/2$ to expose the firm that produces relatively more to less risk (i.e. increasing Δ_1 when $Q_1^H > Q^H/2$, and decreasing it otherwise) has a positive impact on profits through reducing aggregate risk premia. The optimal Δ_1 with intermediate risk aversion is thus bound away from the symmetric outcome. In terms of the optimal Q_1^H, with risk neutrality it has no impact on profits, but with even a small amount of risk aversion the upstream firm again uses it to reduce risk premia.[11] Formally speaking, we show that the signs of the partial derivatives of upstream profits in a neighborhood around symmetric contracts in figure 1b hold for any level of risk aversion.

Rather than being a knife-edge case, the linear demand condition should be understood as implying that partial exclusion arises subject to a bound on the curvature of the demand function. It is a technical condition in the sense that linearity is useful for completing the proof of proposition 3; we leave open the question of how substantial curvature in demand affects the optimal distribution for small values of a.

Combining the results so far together also gives a global prediction on how the degree of risk aversion affects the extent to which optimal distribution contracts are asymmetric. We know from proposition 1 that full symmetry is optimal for $a = 0$. Proposition 3 then shows that as we increase a, both firms continue to produce, but one firm produces more than another. Finally, proposition 2 shows that as a passes a critical threshold, one firm stops producing altogether. So with linear demand, a prediction of the model is that higher levels of downstream risk aversion should be associated with more asymmetry. Intuitively, this is because the risk premium begins to dominate the efficiency gains of output smoothing.

3.3 Limited liability

So far, we have focused on risk aversion as the mechanism that forces the upstream firm to leave profits to downstream firms with competition, but we now show that similar forces arise with limited liability. Instead of assuming that downstream firms are risk averse, we now assume they are risk neutral ($a = 0$) but enjoy limited liability and cannot be forced to absorb losses. Absent limited liability, the participation constraints allow firm i profits to be negative when facing a low-cost competitor and positive when facing a high-cost competitor so long as the expected utility of both events provides a payoff equivalent to leaving the market. In practice limited-liability constraints might make this infeasible. To incorporate these into the model, we follow the approach of Demougin and Garvie (1991) and introduce non-negativity constraints on profits, which we analyze in section

[11] Notice that when $Q_i^H > Q_j^H$, then $Q_i(c) > Q_j(c)$ by definition. Also, $\Delta_i > \Delta_j$ implies $Q_i(0) - Q_i^H > Q_j(0) - Q_j^H$ by definition, or $Q_i(0) - Q_j(0) > Q_i^H - Q_j^H$. So $Q_i^H > Q_j^H$ and $\Delta_i > \Delta_j$ together imply partial exclusion.

3.3. More specifically, we replace the PC constraints in (3) with

$$\min\{\pi_i(c_i, 0, c_i), \pi_i(c_i, c, c_i)\} \geq 0 \qquad (LL)$$

for each c_i. One can interpret these as *ex post* participation constraints that allow downstream firms to exit the market and receive a zero payoff after observing the realization of profits. In contrast, the PC constraints in (3) are interim participation constraints that allow exit after a negative cost shock, but not after a negative revenue shock induced by an efficient competitor.

We begin by stating the main result with limited liability.

Proposition 4 *Under limited downstream liability, there exists an $r' > 0$ such that full exclusion is optimal for all $r < r'$.*

To see the negative effect of competition, recall that the optimal contract with risk neutrality and unlimited liability featured (1) $\Delta^* > 0$, (2) $\Delta_1^* = \Delta_2^*$, and (3) binding participation constraints for high-cost firms. The fact that participation constraints bind and $\Delta_i > 0$ means that high-cost firms earn a profit when facing a high-cost competitor and lose money when facing a low-cost competitor, such that on average profits are zero. Clearly this contract violates the limited liability constraints (LL).

Instead of satisfying high-cost firms' participation constraints, under limited liability the upstream firm instead ensures that they earn zero profit when meeting an efficient competitor, i.e. $\pi_i(c, 0, c) = 0$. But this in turn implies that high-cost firms must earn positive profit when meeting an inefficient competitor. When the probability of efficient firms r is sufficiently low, paying out this surplus to maintain competition is not optimal, and the upstream firm again offers exclusive contracts. Within this region, we also show that the optimal exclusive contract coincides with that offered with high risk aversion.[12]

4 Extensions

In this section, we explore a few extensions of the baseline model. We consider situations in which the upstream firm contracts ex ante with downstream firms and costs are correlated; the upstream firm can choose to eliminate competition with production-only contracts rather than exclusion; and the upstream firm cannot offer different contracts to downstream firms. In each extension, we continue to find conditions under which exclusive contracts are optimal.

[12]These are the values that solve $\text{MR}(Q^{H*} + \Delta^*) = 0$ and $(1 - r)\text{MR}(Q^{H*})$.

4.1 Ex ante contracting with correlated shocks

Up until now all the uncertainty in the model has arisen from each downstream firm not knowing its competitor's type, whereas in reality firms might also not know what the realization of their own shock is going to be at the time they agree to trade with the upstream firm. To capture this feature, we introduce the following, alternative timing of the game.

The upstream firm posts publicly observable contracts as in the baseline model. Each downstream firm then chooses whether to accept its contract, or reject and get 0. After this decision, all downstream firms draw c_i, which they privately observe. All downstream firms that accepted their contracts at the second stage choose Q_i and commit to pay the corresponding transfer $T_i(Q_i)$. Finally, firms produce output, the market clears, profits are realized, and downstream firms pay the transfer to the upstream firm.

We also generalize the model by assuming that the correlation between c_1 and c_2 is $\rho \in [0, 1)$. In its timing and information assumptions, this alternative model is nearly identical to Rey and Tirole (1986), except that they take $\rho = 1$. All that our alternative model requires is that there be *some* interim uncertainty, which we view as realistic. $\rho < 1$ (but potentially very close to 1) essentially requires there to be an arbitrarily small idiosyncratic component of the *ex ante* uncertainty that downstream firms face. For example, firms may face a common demand shock and then a small probability of an individual shock to the marginal product of a production input.

With incentive compatible menus, firm i faces the *ex ante* compound lottery[13]

$$\mathfrak{L}_i = [L_i(0 \mid 0), L_i(c \mid c); (r, 1 - r)] \,. \tag{10}$$

M's new problem can be expressed as

$$\max_{\{Q_i(c_i),T_i(c_i)\}_{i=1,2;c_i\in\{0,c\}}} \sum_{i=1}^{2} rT_i(0) + (1 - r)T_i(c) \text{ such that} \tag{11}$$

$$U[\mathfrak{L}_i] \geq 0 \tag{PC}$$

$$U[L_i(c_i \mid c_i)] \geq U[L_i(c_j \mid c_i)] \text{ for } c_j \neq c_i \tag{IC}$$

$$\overline{Q} \geq Q_i(c_i) \geq 0. \tag{QQ}$$

The participation constraints in this problem do not depend on a firm's cost type. The following result shows that the timing and informational assumptions made in the baseline case are innocuous.

Proposition 5

[13]Whenever $\rho > 0$, the probabilities in the interim lotteries are computed using firm i's posterior beliefs on the distribution of c_j conditional on c_i.

1. *With downstream risk neutrality:* $\Delta^* > 0$ *and* $\Delta_1^* = \Delta_2^* = \frac{\Delta^*}{2}$.

2. *With downstream infinite risk aversion: whenever* $r < 1 - \frac{c}{P(0)}$ *the optimal contracts are such that* $\Delta_i^* > 0$, $Q_i^{H*} > 0$, *and* $\Delta_j^* = Q_j^{H*} = 0$ *for some* $i = 1, 2$ *and* $j \neq i$.

The threshold value of r in part 2 of proposition 5 guarantees that the upstream firm wishes to contract positive high-cost output in aggregate with infinite risk aversion.

The basic difference between this timing and that of the baseline model is the absence of information rents, but because these were invariant to the distribution of aggregate output between firms, their elimination does not materially affect the optimality of exclusion. As for the correlation coefficient, whenever there is a positive probability of two firms having different cost realizations, the upstream firm gains from dealing with all efficient cost types, while infinitely risk averse firms compute their expected utility of the interim lottery under the worst case scenario of meeting the efficient competitor, which is independent of ρ.

4.2 Revenue-sharing (or production-only) contracts

We have argued that risk aversion and limited liability lead to asymmetric outputs among identical downstream firms, and in some cases to full exclusion. These outcomes clearly hurt the upstream firm relative to a situation where firms are risk neutral or not shielded from liability, and one may wonder why the upstream firm does not seek to reduce or eliminate the frictions through reducing the exposure to competition.

A seemingly straightforward way for the upstream firm to eliminate the negative effects of downstream competition is to pay downstream firms to produce output, but then itself collect the revenue from selling the output (a situation that may also be interpreted as vertical integration between the manufacturer and the retailers). This arrangement shields the downstream firms from the negative revenue shock associated with meeting an efficient competitor, and allows the upstream firm to replicate the payoff it obtains from contracting with two risk-neutral firms. As we describe below, though, different contractual arrangements also change the total value of information rents enjoyed by downstream firms.

The specific setup we analyze in this section is one in which the upstream firm offers an exclusive contract under limited liability as described in section 3.3, but has the option to offer revenue-sharing contracts as described above. In this situation, the upstream firm's problem becomes

$$\begin{aligned}
\max_{\{Q_i(c_i),T_i(c_i)\}_{i=1,2;c_i\in\{0,c\}}} \quad & r^2\left[T_1(0)+T_2(0)+(Q^H+\Delta)P(Q^H+\Delta)\right]+ \\
& r(1-r)\left[T_1(0)+T_2(c)+(Q^H+\Delta_1)P(Q^H+\Delta_1)\right]+ \\
& r(1-r)\left[T_1(c)+T_2(0)+(Q^H+\Delta_2)P(Q^H+\Delta_2)\right]+ \\
& (1-r)^2\left[T_1(c)+T_2(c)+Q^HP(Q^H)\right] \text{ such that} \qquad (12) \\
-c_iQ_i(c_i)-T_i(c_i)\geq 0 & \qquad (PC) \\
-c_iQ_i(c_i)-T_i(c_i)\geq -c_iQ_i(c_j)-T_i(c_j) & \text{ for } c_j\neq c_i. \qquad (IC)
\end{aligned}$$

Here the upstream firm's objective directly includes output since it collects sales revenue from downstream production. The transfer payments will clearly now be negative: the upstream firm pays downstream firms for production. Following the same arguments as in the proof of lemma 1, we can show that without loss of generality the optimal transfers can be written as $T_i(c)=T_i(0)=-cQ_i(c)$. When plugged back into the upstream objective above, we obtain precisely the same objective function as in (6).

Our question of interest is to compare outcomes in the case with limited liability and exclusive contracts with those under the solution to (12). To make the comparison tractable, we will assume linear demand $P(Q)=1-Q$. As we prove in the appendix, the optimal exclusive contract offered under limited liability takes the form (for the single firm that produces)[14]

$$Q^{LL}(0)=\frac{1}{2},\ Q^{LL}(c)=\frac{1-c}{2}-\frac{r}{1-r}\frac{c}{2}$$

while the optimal contract under revenue sharing induces each of the two downstream firms to produce

$$Q^{RS}(0)=\frac{1+c}{4},\ Q^{RS}(c)=\frac{1-c}{4}-\frac{r}{1-r}\frac{c}{2}.$$

An initial observation is that

$$rQ^{LL}(0)+(1-r)Q^{LL}(c)=2[rQ^{RS}(0)+(1-r)Q^{RS}(c)]=\frac{1-c}{2},$$

so that the expected output level remains the same in both cases. Instead, as we have emphasized throughout the paper, the *second-moment* effects arising from the different distributions of output across cost types are the important ones. The following result summarizes the resulting welfare impacts.

Proposition 6 *In comparison with the optimal exclusive contract under limited liability,*

[14]According to proposition 4, exclusive contracts are optimal with limited liability for sufficiently low r. With linear demand, the bound in proposition 4 is $r'=\frac{1-c}{1+2c}$, which can be made arbitrarily close to 1 by taking c small enough.

the optimal contract under revenue sharing generates:

1. *Higher upstream profits.*
2. *Lower downstream profits.*
3. *Higher consumer surplus.*

Moreover, joint producer surplus is higher if and only if $r < 0.5$ *while overall surplus is higher if and only if* $r < 0.75$.

Here profits and consumer surplus are computed ex ante, i.e. by taking expectations with respect to downstream shocks.

The fact that upstream profits are higher with two firms is straightforward given the discussion of our baseline results. On the other hand, so far we have not discussed the impact of the upstream firm's choices on downstream profits.[15] Proposition 6 shows that exclusive contracts increase these. Downstream profits are generated by low cost firms' enjoying information rents, and these increase in the production level of high-cost firms. When the upstream firm contracts two firms rather than one, the expected output of high-cost firms decreases and so too do information rents. This effect of competition on downstream profits is increasing in r, the probability of drawing a low cost. Finally, two firms increase consumer surplus by widening the spread between high- and low-cost firm production—consumer surplus is $\frac{Q^2}{2}$, a convex function, so this improves welfare. Intuitively, consumers strongly value large output realizations, which two firms are more likely to provide than one.

The final part of proposition 6 concerns the combined welfare effects. When high-cost firms make up the majority of the population ($r < 0.5$), joint producer surplus (the sum of upstream and downstream profits) is lower with exclusive contracts, but otherwise the negative effect on downstream profits dominates and surplus is higher under an exclusive contract. When one also considers consumer surplus, two firms dominate for a larger range of parameters ($r < 0.75$), but the impact on downstream profits can still be large enough to make exclusive contracts optimal for overall welfare.

In terms of the incentive for the upstream firm to remove the limited-liability constraints that compel it to offer exclusive contracts, we can draw two conclusions. First, if it considers just itself, it will always have an incentive to do so if it can. Second, if some mechanism exists for it to internalize the effect of removing limited liability on downstream firms' profits, it will not do so when low-cost firms are sufficiently common. Such a mechanism might take the form of simple side payments among producers prior to the start of production and the shock realizations. These observations in the context

[15] For a recent paper which looks at how revenue-sharing contracts shift rents in the supply chain, see Johnson (2017).

of our simple example clearly do not form a theory that endogenizes the presence of limited-liability constraints. Rather we make the point that asymmetric outcomes, while harmful to the upstream firm, are not necessarily incompatible with maximizing total producer surplus. Moreover, the choice of the upstream firm to offer exclusive contracts is not necessarily in line with maximizing total welfare.

More broadly speaking, another relevant concern for upstream firms is moral hazard. There is a large literature, summarized in Blair and Lafontaine (2011), that models upstream firms as risk-neutral principals and downstream retailers as risk-averse agents who make investments in quality. As is well known in the literature (Holmström 1979), insurance comes at the cost of incentives and would be expected to reduce the amount of investment.

4.3 Standardized contracts

The finding that in some cases the upstream firm chooses exclusive contracts to the detriment of social welfare suggests a possible role for regulatory intervention. Since exclusion in our model may arise because of discriminatory offers or through an explicit exclusive clause, one could imagine a possible policy intervention prohibiting exclusive clauses and mandating contracts that are standardized across (i.e., do not discriminate between) downstream firms. More formally, the upstream firm would face the requirement that $T_1(Q_1) = T_2(Q_2)$ whenever $Q_1 = Q_2$. One might imagine that introducing this requirement would force the upstream firm to deal with both retailers, but in this section we show that such contracts do not in fact rule out its ability to replicate the optimal exclusive outcome with discrimination as an equilibrium outcome under (possibly very mild) conditions.

When the upstream firm can offer different contracts to different downstream firms, we know that under infinite risk aversion (as long as r is sufficiently small) exclusion of one of them is uniquely optimal. The question is whether the upstream firm is able to replicate this outcome when it is restricted to offering the same standardized contract to both firms.[16] More precisely, the upstream firm would like to implement the following equilibrium outcome:

1. It offers both firms the same menu of contracts $\{(T(0), Q(0)), (T(c), Q(c)), (0, 0)\}$, with $Q(0) = Q^{H*} + \Delta^*$, $Q(c) = Q^{H*}$, $T(0) = (Q^{H*} + \Delta^*)P(Q^* + \Delta^*) - cQ^{H*}$, and $T(c) = Q^{H*}P(Q^{H*}) - cQ^{H*}$, where Q^{H*} and Δ^* are the optimal values found for an exclusive contract.

[16] This is similar to Segal (2003), who examines when the optimal allocation under bilateral contracting with individualized contracts can be replicated with standardized contracts.

2. One firm (say 1) chooses $(T(0), Q(0))$ if low cost and $(T(c), Q(c))$ if high cost; and the other firm (say 2) chooses $(0, 0)$ for both cost types.

For this to be an equilibrium outcome, each player must find the candidate strategy to be optimal given the choices of the others. For the upstream firm, optimality follows immediately from the fact that this contract reproduces the optimal outcome obtained under less restrictive assumptions on its strategies (it cannot achieve higher profits when it is obliged to use standardized contracts than when it can discriminate).

In the case of firm 1, the choice is also optimal given that firm 2 chooses not to participate in the market: since $(0, 0)$ effectively amounts to exclusion, we know that the participation and incentive constraints for firm 1 are satisfied when firm 2 does not sell.

Therefore, we only need to confirm whether firm 2 prefers not to participate (i.e., chooses $(0, 0)$) given that firm 1 chooses $(T(0), Q(0))$ if low cost and $(T(c), Q(c))$ if high cost. Under infinite risk aversion, this is true when the following incentive constraints are satisfied:

$$0 \geq Q^{H*} P\left(2Q^{H*} + \Delta^*\right) - cQ^{H*} - T(c) \qquad (IC_{HH})$$

$$0 \geq \left(Q^{H*} + \Delta^*\right) P\left(2Q^{H*} + 2\Delta^*\right) - c\left(Q^{H*} + \Delta^*\right) - T(0) \qquad (IC_{HL})$$

$$0 \geq Q^{H*} P\left(2Q^{H*} + \Delta^*\right) - T(c) \qquad (IC_{LH})$$

$$0 \geq \left(Q^{H*} + \Delta^*\right) P\left(2Q^{H*} + 2\Delta^*\right) - T(0), \qquad (IC_{LL})$$

where IC_{HH} and IC_{HL} refer to the possible deviations of high-cost firm 2 (it might want to pick $(T(c), Q(c))$ or $(T(0), Q(0))$, respectively); and similarly IC_{LH} and IC_{LL} refer to the possible deviations of low-cost firm 2. After substitution, the four ICs become:

$$0 \geq Q^{H*}\left[P\left(2Q^{H*} + \Delta^*\right) - P(Q^{H*})\right] \qquad (IC_{HH})$$

$$0 \geq (Q^{H*} + \Delta^*)\left[P\left(2Q^{H*} + 2\Delta^*\right) - P\left(Q^{H*} + \Delta^*\right)\right] - c\Delta^* \qquad (IC_{HL})$$

$$0 \geq Q^{H*}\left[P\left(2Q^{H*} + \Delta^*\right) - P(Q^{H*})\right] + cQ^{H*} \qquad (IC_{LH})$$

$$0 \geq \left(Q^{H*} + \Delta^*\right)\left[P\left(2Q^{H*} + 2\Delta^*\right) - P\left(Q^{H*} + \Delta^*\right)\right] + cQ^{H*}. \qquad (IC_{LL})$$

Because the demand function is assumed to be decreasing, the first two constraints are always satisfied, so we are left with the last two ICs. We find that

Proposition 7 *There exists a $\bar{c} > 0$ such that for $c \leq \bar{c}$, the upstream firm is able to implement the (optimal) exclusionary outcome by making use of standardized contracts.*

Of course, implementing the exclusive contract in this way requires downstream firms, which are ex ante symmetric, to coordinate on an asymmetric equilibrium in which one produces and one does not. In this sense, explicit contractual discrimination is arguably a more straightforward means through which the upstream firm can generate exclusion in equilibrium.

Another case of interest is when the upstream firm would like to implement the optimal exclusive contract when downstream firms are risk neutral but face limited liability. In this situation, the incentive constraints for the non-producing firm 2 are stricter than with infinite risk aversion because we need to check that there is no deviation that yields strictly positive profit for any possible realization of firm 1's cost type. This is because under limited liability firm 2 can 'walk away' from losses it may suffer when deviating and producing. Still, one can show that for sufficiently low c, the same result as in proposition 7 holds and the upstream firm does not suffer from being restricted to standardized contracts.

One of the implications of this analysis is that it would be meaningless to discuss the desirability of policies such as prohibiting the manufacturer to make use of exclusive clauses or discriminatory terms in its contract offers: by resorting to a sufficiently rich menu of options, the manufacturing can, in many cases, achieve the same outcomes as in explicitly exclusive or discriminatory offers. For example, under linear demand $P(Q) = 1 - Q$, one can show that the optimal exclusive contract is implementable with infinite risk aversion under standardized contracts whenever $c < \overline{c} = \frac{1}{2}$.

4.4 Relaxing other assumptions

In this section we briefly and informally discuss the role of other assumptions we make in the model.

Number of downstream firms and number of cost types. Our baseline model has two firms each with two cost types. One can show (Hansen and Motta 2012) that in a model with N firms each with M possible (symmetrically distributed) cost types, the optimal contract is fully exclusive under infinite risk aversion (and, by continuity, for high enough a) as long as the upstream firm wishes to contract positive output from the second-most efficient cost type. A separate issue is the effect an increased number of firms has on the trade-off between efficient production and risk for a fixed, finite level of risk aversion. One intuition is that exclusion becomes less likely as the number of downstream firms increases. Our results on exclusion rely on the upstream firm wishing to contract a positive output level from inefficient firms, but with more and more downstream firms the need to offer inefficient firms a share of the market decreases.

Price competition. We have assumed that downstream firms compete in quantities and sell homogeneous goods. With price competition, if firms sell homogeneous goods then the Nash equilibrium of the pricing game with private information on costs involves mixed strategies that make characterizing the optimal contract intractable. If we instead assume that retailers compete in prices but are sufficiently differentiated for all cost types to produce positive output in equilibrium, then one can easily show the same mechanism from our baseline case applies: competition exposes retailers to variability in their profits due to the variation in their competitor's costs. This increases the incentives for the upstream firm to offer exclusive contracts.

Asymmetric Downstream Firms. Throughout the paper, we have assumed that downstream firms are fully symmetric in order to emphasize that risk aversion and limited liability alone can generate asymmetric outcomes. If downstream firms were fundamentally asymmetric, then a reasonable conjecture would be that exclusive contracting would become even more attractive for the upstream firm. For example, suppose one downstream retailer had lower expected costs than another. Even under risk neutrality, the upstream firm would like to contract more output from this firm than its competitor. The introduction of risk aversion would then reënforce the degree to which the upstream firm relied on the more efficient retailer to serve the market. Alternatively, if the downstream firms differed in their levels of risk aversion, one would imagine that the upstream firm would sell more via the less risk-averse firm.

5 Conclusion

This paper identifies a new rationale for using exclusivity provisions: when firms compete downstream, and do not perfectly observe one another's productivity shocks, competition generates uncertainty, leading risk-averse (limited-liable) agents to require a risk premium (additional surplus). To save on these costs, the upstream firm sometimes prefers to deal exclusively with one firm, and more generally offers asymmetric contracts in many cases.

As mentioned in the introduction, an alternative story for observing exclusion is that upstream firms suffer from commitment problems. In franchise networks, the main commitment problem is *encroachment* whereby franchisors allow new franchisees to open outlets in areas previously successfully developed by established franchisees. Exclusive territories are often cited as a means of reassuring franchisees that such encroachment will not take place. Blair and Lafontaine (2011) argue that encroachment should be more problematic the larger the network, and then contrast this with the empirical evidence. One study found that only 26 of the largest 50 restaurant franchisors in the US offered an exclusive territory, compared with an overall incidence for restaurant franchisors of around 75%. On the other hand, Azoulay and Shane (2001) collected a dataset of newly founded franchises across a variety of industries, and report that 84% offered exclusive territories, whereas the cross-industry incidence in the US is around 73%. Hence in the networks where encroachment should be seemingly less of a problem, exclusion is *more* likely to be observed. An explanation of this observation is that franchisees in new networks face greater uncertainty than in existing networks, and that exclusion protects them in part against the concomitant risk.

In section 4, we provide several robustness checks on the basic results of this paper. The most notable one is the introduction of ex ante participation constraints in place of interim ones, as well as correlation between shock realizations. This brings the model very close to that of Rey and Tirole (1986), who adopt a setup with perfect correlation (also compatible with downstream firms' facing a common shock in addition to an idiosyncratic one). We show that *any* imperfect correlation leads to the optimality of full exclusion with infinite risk aversion, in direct contrast to Rey and Tirole (1986). This underscores the point that even small amounts of uncertainty can have dramatic impacts on the distribution of output across firms.

One important issue is why the upstream firm does not offer more insurance to downstream firms, as this would clearly improve its payoff. For example, one possibility would be to pay downstream firms a fixed amount to produce, but then collect the revenue itself. However, we have shown that under certain conditions producer surplus (the sum of upstream and downstream profits) is higher with exclusive contracts than with revenue-sharing contracts. Hence if there is a mechanism through which the upstream firm internalizes the effect of providing insurance to downstream firms, such as ex-ante bargaining, it will not necessarily choose to do so.

Unlike in the literature on commitment, in our setup the upstream firm need not use explicit exclusive clauses to induce exclusive outcomes. This makes regulating exclusion in such markets challenging. In some situations, the upstream firm can even implement exclusive outcomes through uniform contracts offered to potential entrants who then self select into producing or not. We thus view the main contribution of the paper as providing a positive account of the choice of upstream firms to induce asymmetric downstream outcomes.

More broadly, our basic logic offers a general reason why a principal may endogenously restrict the number of agents with whom it wants to deal. Whenever the payoff of one agent depends on the actions or the types of other agents, and there is imperfect information, the introduction of competition will oblige the principal to pay a risk premium whenever agents are risk averse. To save on these, the principal may prefer to contract with a strict subset of the potential agents. This same mechanism should hold in very different settings, such as in a moral hazard model where agents are paid according to relative performance schemes.

References

Anand, B. N. and Khanna, T. (2000). The Structure of Licensing Contracts. *Journal of Industrial Economics*, 48(1):103–35.

Asplund, M. (2002). Risk-averse firms in oligopoly. *International Journal of Industrial Organization*, 20(7):995–1012.

Azoulay, P. and Shane, S. (2001). Entrepreneurs, Contracts, and the Failure of Young Firms. *Management Science*, 47(3):337–358.

Banal-Estañol, A. and Ottaviani, M. (2006). Mergers with Product Market Risk. *Journal of Economics & Management Strategy*, 15(3):577–608.

Blair, R. D. and Lafontaine, F. (2011). *Economics of Franchising*. Cambridge University Press, first paperback edition.

Dasgupta, S. and Spulber, D. F. (1989). Managing procurement auctions. *Information Economics and Policy*, 4(1):5–29.

Demougin, D. M. and Garvie, D. A. (1991). Contractual Design with Correlated Information under Limited Liability. *RAND Journal of Economics*, 22(4):477–489.

Dequiedt, V. and Martimort, D. (2015). Vertical Contracting with Informational Opportunism. *American Economic Review*, 105(7):2141–82.

Hansen, S. and Motta, M. (2012). Vertical Exclusion with Endogenous Competiton Externalities. CEPR Discussion Papers 8982, C.E.P.R. Discussion Papers.

Hart, O. and Tirole, J. (1990). Vertical Integration and Market Foreclosure. *Brookings Papers on Economic Activity (Microeconomics)*, 1990:205–286.

Holmström, B. (1979). Moral Hazard and Observability. *Bell Journal of Economics*, 10(1):74–91.

Johnson, J. P. (2017). The Agency Model and MFN Clauses. *Review of Economic Studies*, 84(3):1151–1185.

Kaufmann, P. J. and Lafontaine, F. (1994). The Costs of Control. *Journal of Law and Economics*, 37(2):417–53.

Laffont, J.-J. and Tirole, J. (1987). Auctioning Incentive Contracts. *Journal of Political Economy*, 95(5):921–37.

Lafontaine, F. (1992). How and Why do Franchisors do What They do: A Survey Report. In Kaufmann, P. J., editor, *Sixth Annual Proceedings of the Society of Franchising*.

Lafontaine, F. and Shaw, K. L. (1999). The Dynamics of Franchise Contracting: Evidence from Panel Data. *Journal of Political Economy*, 107(5):1041–1080.

Lafontaine, F. and Slade, M. (2008). Empirical Assessment of Exclusive Contracts. In Buccirossi, P., editor, *Handbook of Antitrust Economics*. MIT Press, Cambdrige.

McAfee, R. P. and McMillan, J. (1986). Bidding for Contracts: A Principal-Agent Analysis. *RAND Journal of Economics*, 17(3):326–338.

McAfee, R. P. and McMillan, J. (1987). Competition for Agency Contracts. *RAND Journal of Economics*, 18(2):296–307.

McAfee, R. P. and Schwartz, M. (1994). Opportunism in Multilateral Vertical Contracting: Nondiscrimination, Exclusivity, and Uniformity. *American Economic Review*, 84(1):210–30.

Nocke, V. and Thanassoulis, J. (2014). Vertical Relations Under Credit Constraints. *Journal of the European Economic Association*, 12(2):337–367.

Rey, P. and Tirole, J. (1986). The Logic of Vertical Restraints. *The American Economic Review*, 76(5):921–939.

Rey, P. and Tirole, J. (2007). A Primer on Foreclosure. In Armstrong, M. and Porter, R., editors, *Handbook of Industrial Organization*, pages 2145–2220. North-Holland.

Riordan, M. H. and Sappington, D. E. M. (1987). Awarding Monopoly Franchises. *American Economic Review*, 77(3):375–87.

Scott Morton, F. (2012). Contracts that Reference Rivals. Speech delivered at Georgetown University Law Center, April 5, 2012. Available at http://www.justice.gov/atr/file/518971/download.

Segal, I. (1999). Contracting With Externalities. *The Quarterly Journal of Economics*, 114(2):337–388.

Segal, I. (2003). Coordination and discrimination in contracting with externalities: divide and conquer? *Journal of Economic Theory*, 113(2):147–181.

Sundaram, R. K. (1996). *A First Course in Optimization Theory*. Cambridge University Press.

A Proofs

In all proofs in which we derive optimal downstream outputs, we ignore the constraint that output is less than $\overline{Q}$. In all cases, optimal output is finite, so by taking $\overline{Q}$ large enough, we can safely ignore it. Its sole role is to guarantee the compactness of the domain over which the upstream firm maximizes in the proof of proposition 2.

A.1 Proof of Lemma 1 / Upstream Objective Function

Proof. Suppose an agent faces the lottery $[(w, w-L), (1-r, r)]$ and has CARA utility. The certainty equivalent C is defined by

$$\exp(-aC) = (1-r)\exp(-aw) + r\exp(-a(w-L))$$

which after algebraic manipulations gives $C = w - \Gamma(L,a,r)$ where $\Gamma(L,a,r) \equiv \frac{\ln[(1-r)+r\exp(aL)]}{a}$. Applying this expression to the lotteries faced by downstream firms in the model gives

$$\text{CertRev}_i(\hat{c}_i) =$$
$$Q_i(\hat{c}_i)P[Q_i(\hat{c}_i)+Q_j(c)] - \Gamma\left(Q_i(\hat{c}_i)\left\{P[Q_i(\hat{c}_i)+Q_j(c)] - P[Q_i(\hat{c}_i)+Q_j(0)]\right\}, a, r\right). \quad \text{(A.1)}$$

Incentive compatibility for the low-cost firm and participation of the high-cost firm imply participation of the low-cost firm since

$$\text{CertRev}_i(0) - T_i(0) \geq \text{CertRev}_i(c) - T_i(c) > \text{CertRev}_i(c) - Q_i(c)c - T_i(c) \geq 0.$$

So we can drop the low-cost participation constraint from the program (3). Consider now the relaxed problem in which we only consider the participation constraint of the high-cost firm and the incentive-compatibility constraint of the low-cost firm (i.e. we drop the IC constraint of the high-cost firm). This relaxed program is

$$\max_{\{Q_i(c_i),T_i(c_i)\}_{i=1,2;c_i\in\{0,c\}}} \sum_{i=1}^{2} rT_i(0) + (1-r)T_i(c) \text{ such that} \quad \text{(A.2)}$$
$$\text{CertRev}_i(c) - Q_i(c)c - T_i(c) \geq 0$$
$$\text{CertRev}_i(0) - T_i(0) \geq \text{CertRev}_i(c) - T_i(c)$$
$$\overline{Q} \geq Q_i(c_i) \geq 0.$$

First notice that at no solution could there be a slack participation constraint for the high-cost firms since otherwise the upstream firm could increase $T_i(c)$, raise profit, and continue to satisfy the low-cost IC constraint. Second, at no solution could there be a slack IC constraint since otherwise the upstream firm could increase $T_i(0)$ and raise profit without affecting high-cost participation. Third, the ignored IC constraint for the high-cost firms can be written as

$$c[Q_i(0) - Q_i(c)] \geq [T_i(c) - T_i(0)] + [\text{CertRev}_i(0) - \text{CertRev}_i(c)].$$

In the solution to the relaxed program, the right-hand side of this expression is zero since the IC constraint for the low-cost firm is binding. So, the solution to the relaxed program is also the solution to the original program provided that $Q_i(0) \geq Q_i(c)$.

The maximization problem described in the lemma is obtained by substituting in for $T_i(0)$ and $T_i(c)$ and imposing the condition $Q_i(0) \geq Q_i(c)$. ■

We now derive the analytical expression for the upstream firm's objective function we use in the remaining formal results. Let $Q_i^H \equiv Q_i(c)$, $\Delta_i \equiv Q_i(0) - Q_i^H$, $Q^H \equiv Q_1^H + Q_2^H$, and $\Delta \equiv \Delta_1 + \Delta_2$. We can then express the choice variables in the maximization problem in terms of the vector $\mathbf{S} = (Q_1^H, Q^H, \Delta_1, \Delta)$, which must satisfy the constraints $Q^H \geq Q_1^H \geq 0$ and $\Delta \geq \Delta_1 \geq 0$. Plugging into (A.1), we obtain

$$\begin{aligned}
&\text{CertRev}_i(c) = Q_i^H P(Q^H) - \Gamma\left\{Q_i^H \left[P(Q^H) - P(Q^H + \Delta_j)\right], a, r\right\} \text{ and} \\
&\text{CertRev}_i(0) = (Q_i^H + \Delta_i)P(Q^H + \Delta_i) - \Gamma\left\{(Q_i^H + \Delta_i)\left[P(Q^H + \Delta_i) - P(Q^H + \Delta)\right], a, r\right\}
\end{aligned}$$

After applying the substitutions $Q_2^H = Q^H - Q_1^H$ and $\Delta_2 = \Delta - \Delta_1$, the upstream firm's objective function becomes[17]

$$\begin{gathered}
\pi(\mathbf{S}, a, r, c) = \\
(1-r)Q^H P\left(Q^H\right) + r\left[\left(Q_1^H + \Delta_1\right) P\left(Q^H + \Delta_1\right) + \left(Q_2^H + \Delta - \Delta_1\right) P\left(Q^H + \Delta - \Delta_1\right)\right] - \\
(1-r)\Gamma\left\{Q_1^H\left[P\left(Q^H\right) - P\left(Q^H + \Delta_2\right)\right]\right\} - (1-r)\Gamma\left\{\left(Q^H - Q_1^H\right)\left[P\left(Q^H\right) - P\left(Q^H + \Delta_1\right)\right]\right\} - \\
r\Gamma\left\{\left(Q_1^H + \Delta_1\right)\left[P\left(Q^H + \Delta_1\right) - P\left(Q^H + \Delta\right)\right]\right\} - \\
r\Gamma\left\{\left(Q^H - Q_1^H + \Delta - \Delta_1\right)\left[P\left(Q^H + \Delta - \Delta_1\right) - P\left(Q^H + \Delta\right)\right]\right\} - cQ^H. \qquad \text{(A.3)}
\end{gathered}$$

By l'Hôpital's Rule we obtain the relationships

$$\begin{aligned}
&\lim_{a \to 0} \Gamma(L, a, r) = \lim_{a \to 0} \frac{rL\exp(aL)}{r\exp(aL) + 1 - r} = rL \\
&\lim_{a \to \infty} \Gamma(L, a, r) = \lim_{a \to \infty} \frac{rL\exp(aL)}{r\exp(aL) + 1 - r} = \lim_{a \to \infty} \frac{L}{1 + \frac{1-r}{r}\exp(-aL)} = L.
\end{aligned}$$

In other words, the limit of certainty equivalent revenue as $a \to 0$ corresponds to risk neutrality, as certainty equivalent income is expected income. The limit as $a \to \infty$ corresponds to infinite risk aversion, as the payoff from a lottery is its worst realization. We extend the definition of $\pi(\mathbf{S}, a, r, c)$ to $a = 0$ with expression (6) in the main text, and to $a = \infty$ with (9). Given the limit results, $\pi(\mathbf{S}, a, r, c)$ defined in this way in continuous in $a \in \mathbb{R}_+ \cup \infty$.

[17]Here for notational compactness we have dropped the a and r parameters from the Γ functions.

A.2 Proof of Lemma 2

Proof. The proof for why $\Delta^* > 0$ proceeds exactly as in the proof of proposition 1. In particular, note that when $\Delta = 0$ upstream profits from (A.3) become $Q^H \left[P\left(Q^H\right) - c\right]$, which can be obtained with an exclusive contract. But the optimal exclusive contract must have $\Delta > 0$.

Now consider some $\mathbf{S}' = (0, 0, \Delta_1, \Delta)$ for any $0 \le \Delta_1 \le \Delta$. From (A.3) it is easy to check that $\lim_{r\to 0, c\to 0} \pi(\mathbf{S}', a, r, c) = 0$. On the other hand, let $Q^{H\prime} \equiv \arg\max_{Q^H} Q^H \left[P\left(Q^H\right) - c\right]$. Let $\mathbf{S}'' = (Q^{H\prime}, Q^{H\prime}, 0, 0)$. Clearly $\lim_{r\to 0, c\to 0} \pi(\mathbf{S}'', a, r, c)$ is positive, which implies there exists some r^* and c^* such that $\mathbf{S}''$ produces higher profit that $\mathbf{S}'$, meaning that $\mathbf{S}'$ cannot be optimal. ■

A.3 Proof of Proposition 1

Proof. It remains to be shown that the optimal value of Δ is positive. The only other possibility is that $\Delta = 0$ since $\Delta < 0$ would violate incentive compatibility. Suppose that $\Delta = 0$, and let the optimal value of Q^H under this restriction be $Q^{H\prime}$. The total profit of the upstream firm from this solution is $Q^{H\prime}\left[P\left(Q^{H\prime}\right) - c\right]$. Note that this payoff can be obtained with the exclusive contract $Q_1^H = Q^{H\prime}$ and $Q_2^H = 0$. (By assumption $\Delta_1 = \Delta_2 = 0$).

Now consider the upstream firm's choice of the optimal exclusive contract, i.e. a contract in which, without loss of generality, $Q_1^H = Q^H$ and $\Delta_1 = \Delta$. The relevant program is

$$\max_{Q^H \ge 0, \Delta \ge 0} r\left(Q^H + \Delta\right) P\left(Q^H + \Delta\right) + (1-r) Q^H P\left(Q^H\right) - cQ^H. \tag{A.4}$$

The optimal values (Q^{H*}, Δ^*) for this problem solve the first order conditions

$$\text{MR}\left(Q^{H*} + \Delta^*\right) \le 0 \tag{A.5}$$

$$r\text{MR}\left(Q^{H*} + \Delta^*\right) + (1-r)\text{MR}\left(Q^{H*}\right) - c \le 0 \tag{A.6}$$

where (A.5) holds with equality if $\Delta^* > 0$ and (A.6) holds with equality if $Q^{H*} > 0$. These conditions together imply that $\Delta^* > 0$. Suppose not, and that $Q^{H*} = 0$. Then, from (A.5), it must be the case that $\text{MR}(0) \le 0$ which is ruled out by assumption. Suppose not, and that $Q^{H*} > 0$. Then (A.6) gives $\text{MR}\left(Q^{H*}\right) = c > 0$ while (A.5) gives $\text{MR}\left(Q^{H*}\right) < 0$, a contradiction. Since the contracts $Q^H = Q^{H\prime}$ and $\Delta_1 = \Delta = 0$ are within the set of feasible contracts for (A.4) and are not chosen, their optimality is contradicted. ■

A.4 Proof of Proposition 2

Proof. The proof relies on continuity in the upstream firm's objective function and its derivatives. We first establish some relevant notation and properties of the solutions.

Let $\mathbf{S}^*(a, r, c) \subset \mathbb{R}_+^4$ denote the set of solutions to program (3) given a. By arguments in the main text, $S^*(\infty, r, c) = \{(Q^{H*}, 0, \Delta^*, 0), (0, Q^{H*}, 0, \Delta^*)\}$ whenever $Q^{H*} > 0$, i.e. all output is offered to one firm when $a = \infty$. After we replace $Q_1^H = Q^H$ and $\Delta_1 = \Delta$ (or, equivalently,

$Q_2^H = Q^H$ and $\Delta_2 = \Delta$) into the manufacturer's profits in (9), we obtain exactly expression (A.4) from the proof of proposition 1. Since the program is equivalent, the optimal values for Δ and Q^H are described by (A.5) and (A.6), respectively. Combining these implies $Q^{H*} > 0$ if and only if $r < 1 - \frac{c}{MR(0)}$. Moreover, $\Delta^* > 0$ as argued in the proof of proposition 1.

Second, from expression (9)

$$\frac{\partial \pi(\mathbf{S}, \infty, r, c)}{\partial Q_1^H} = (1-r)[P(Q^H + \Delta - \Delta_1) - P(Q^H + \Delta_1)] \gtreqless 0 \Leftrightarrow \Delta_1 \gtreqless \frac{\Delta}{2}.$$

Moreover,

$$\frac{\partial \pi(\mathbf{S}, \infty, r, c)}{\partial \Delta_1} \propto Q^H P'(Q^H + \Delta_1) - Q_1^H [P'(Q^H + \Delta - \Delta_1) + P'(Q^H + \Delta_1)].$$

This is strictly increasing in Q_1^H since $P' < 0$. Moreover, this expression is negative when $Q_1^H = 0$ and positive when $Q_1^H = Q^H$. So, we conclude there exists some $\Theta(\Delta_1) \in (0, Q^H)$ such that $\frac{\partial \pi(\mathbf{S}, \infty, r, c)}{\partial \Delta_1} \gtreqless 0 \Leftrightarrow Q_1^H \gtreqless \Theta(\Delta_1)$. The signs of these derivatives are plotted in figure 1b in the main text.

Since $\pi(\mathbf{S}, a, r, c)$ is continuous and the constrained set of parameters is compact, $\mathbf{S}^*(a, r, c)$ is upper-semicontinuous by the Maximum Theorem (see Sundaram (1996) theorem 9.14 for details). Let V be an open set such that $\mathbf{S}^*(\infty, r, c) \subset V$ and for which, for all $\mathbf{S} \in V \cap D$ either $\frac{\partial \pi(\mathbf{S}, \infty, r, c)}{\partial Q_1^H}, \frac{\partial \pi(\mathbf{S}, \infty, r, c)}{\partial \Delta_1} > 0$ or $\frac{\partial \pi(\mathbf{S}, \infty, r, c)}{\partial Q_1^H}, \frac{\partial \pi(\mathbf{S}, \infty, r, c)}{\partial \Delta_1} < 0$. By upper-semicontinuity there exists some a^1 such that $\mathbf{S}^*(a, r, c) \subset V$ for all $a > a^1$. Moreover, since $\pi(\mathbf{S}, a, r, c)$ has continuous derivatives, for any $\mathbf{S} \in V \cap D$ there exists some $a^2(\mathbf{S})$ such that $\frac{\partial \pi(\mathbf{S}, a, r, c)}{\partial Q_1^H}, \frac{\partial \pi(\mathbf{S}, a, r, c)}{\partial \Delta_1} > 0$ and $\frac{\partial \pi(\mathbf{S}, a, r, c)}{\partial Q_1^H}, \frac{\partial \pi(\mathbf{S}, a, r, c)}{\partial \Delta_1} < 0$ for all $a > a^2(\mathbf{S})$. Hence $\mathbf{S}$ cannot maximize profit unless it is fully exclusive. The proof is completed by taking $\overline{a} \equiv \max\{a^1, \max_{\mathbf{S}} a^2(\mathbf{S})\}$. ■

A.5 Proof of Proposition 3

Proof. The strategy of the proof is as follows. We fix some $Q^H > 0$ and $\Delta > 0$, and then consider the problem of maximizing $\pi(Q_1^H, Q^H, \Delta_1, \Delta, a, r, c)$ with respect to Q_1^H and Δ_1 treating Q^H and Δ as fixed, positive parameters. Denote the derived function as $\pi(Q_1^H, \Delta_1)$. Without loss of generality, we consider contracts in which $Q_1^H \geq Q^H/2$. We show that $\Delta_1 > \Delta/2$ and $Q_1^H > Q^H/2$ is optimal within this class. For notational compactness, we express $\Delta - \Delta_1$ as Δ_2 and $Q^H - Q_1^H = Q_2^H$ in some expressions below in line with the definitions in the main text.

Several properties of the Γ function defined in the proof of lemma 1 are useful in the proof:

1. $$\frac{\partial \Gamma(L, a, r)}{\partial L} = \frac{r \exp(aL)}{1 - r + r\exp(aL)} > 0.$$

2. From the above expression, we clearly have $\frac{\partial \Gamma(L, a, r)}{\partial L} \in (r, 1)$.

3.
$$\frac{\partial^2\Gamma(L,a,r)}{\partial L^2} = \frac{(1-r)ar\exp(aL)}{[1-r+r\exp(aL)]^2} > 0.$$

The first step in the proof is to compute and sign the partial derivatives of $\pi(Q_1^H, \Delta_1)$. From (A.3), we can compute

$$\begin{gathered}\frac{\partial\pi}{\partial Q_1^H} = r\left[P\left(Q^H+\Delta_1\right) - P\left(Q^H+\Delta_2\right)\right] - \\ r\left[P\left(Q^H+\Delta_1\right) - P\left(Q^H+\Delta\right)\right]\Gamma'\left\{\left(Q_1^H+\Delta_1\right)\left[P\left(Q^H+\Delta_1\right) - P\left(Q^H+\Delta\right)\right]\right\} + \\ r\left[P\left(Q^H+\Delta_2\right) - P\left(Q^H+\Delta\right)\right]\Gamma'\left\{\left(Q^H-Q_1^H+\Delta_2\right)\left[P\left(Q^H+\Delta_2\right) - P\left(Q^H+\Delta\right)\right]\right\} - \\ (1-r)\left[P\left(Q^H\right) - P\left(Q^H+\Delta_2\right)\right]\Gamma'\left\{Q_1^H\left[P\left(Q^H\right) - P\left(Q^H+\Delta_2\right)\right]\right\} + \\ (1-r)\left[P\left(Q^H\right) - P\left(Q^H+\Delta_1\right)\right]\Gamma'\left\{\left(Q^H-Q_1^H\right)\left[P\left(Q^H\right) - P\left(Q^H+\Delta_1\right)\right]\right\}\end{gathered}$$

In this expression and those that follow in the proof, $\Gamma'\{X\}$ should be understood as $\frac{\partial\Gamma(L,a,r)}{\partial L}$ evaluated at $L = X$. Notice that since $\Gamma'\{X\}$ is increasing in X, the above expression is monotonically decreasing in Q_1^H. So for each Δ_1 there is a unique optimal value for Q_1^H.

Evaluating this expression with linear demand and simplifying gives

$$\begin{gathered}\Delta_1\left\{r\Gamma'[(Q^H-Q_1^H+\Delta_2)\Delta_1] + (1-r)\Gamma'[(Q^H-Q_1^H)\Delta_1] - r\right\} - \\ \Delta_2\left\{r\Gamma'[(Q_1^H+\Delta_1)\Delta_2] + (1-r)\Gamma'[Q_1^H\Delta_2] - r\right\}.\end{gathered} \tag{A.7}$$

First note that when $\Delta_1 = \Delta_2$ this derivative is zero at $Q_1^H = Q^H/2$.

We now show that whenever $\Delta_1 > \Delta_2$, (A.7) is positive when evaluated at $Q_1^H = Q^H/2$. This implies that the optimal value of Q_1^H when $\Delta_1 > \Delta/2$ is greater than $Q^H/2$. The proof proceeds by the construction of lower bounds. First, by the assumption that $\Delta_1 > \Delta_2$ and monotonicity of Γ', we know that (A.7) is larger than

$$\begin{gathered}\Delta_1\left\{r\Gamma'\left[\left(\frac{Q^H}{2}+\Delta_1\right)\Delta_2\right] + (1-r)\Gamma'\left[\frac{Q^H}{2}\Delta_2\right] - r\right\} - \\ \Delta_2\left\{r\Gamma'\left[\left(\frac{Q^H}{2}+\Delta_1\right)\Delta_2\right] + (1-r)\Gamma'\left[\frac{Q^H}{2}\Delta_2\right] - r\right\}\end{gathered}$$

which itself is larger than (again by monotonicity of Γ')

$$\begin{gathered}\Delta_1\left\{r\Gamma'\left[\frac{Q^H}{2}\Delta_2\right] + (1-r)\Gamma'\left[\frac{Q^H}{2}\Delta_2\right] - r\right\} - \Delta_2\left\{r\Gamma'\left[\frac{Q^H}{2}\Delta_2\right] + (1-r)\Gamma'\left[\frac{Q^H}{2}\Delta_2\right] - r\right\} > \\ \Delta_1\left\{\Gamma'\left[\frac{Q^H}{2}\Delta_2\right] - r\right\} - \Delta_2\left\{\Gamma'\left[\frac{Q^H}{2}\Delta_2\right] - r\right\}.\end{gathered}$$

Finally, we know that the final expression above is positive since $\Delta_1 > \Delta_2$ and $\Gamma' \in (r, 1)$. This implies that whenever $\Delta_1 > \Delta_2$, $Q_1^{H*} > Q^H/2$.

With linear demand, the part of the objective function (A.3) that depends on Δ_1 becomes

$$r[(Q_1^H+\Delta_1)(1-Q^H-\Delta_1)+(Q_2^H+\Delta-\Delta_1)(1-Q^H-\Delta+\Delta_1)]-$$
$$r\Gamma[(Q_1^H+\Delta_1)(\Delta-\Delta_1)]-r\Gamma[(Q_2^H+\Delta-\Delta_1)\Delta_1]-(1-r)\Gamma[Q_1^H(\Delta-\Delta_1)]-(1-r)\Gamma[Q_2^H\Delta_1].$$

The derivative with respect to Δ_1 is

$$r(1-Q^H-Q_1^H-2\Delta_1)-r(1-Q^H-Q_2^H-2(\Delta-\Delta_1))-$$
$$r(\Delta-2\Delta_1-Q_1^H)\Gamma'[(Q_1^H+\Delta_1)(\Delta-\Delta_1)]-r(\Delta-2\Delta_1+Q_2^H)\Gamma'[(Q_2^H+\Delta-\Delta_1)\Delta_1]-$$
$$(1-r)(-Q_1^H)\Gamma'[Q_1^H(\Delta-\Delta_1)]-(1-r)Q_2^H\Gamma'[Q_2^H\Delta_1].$$

which can be re-written as

$$r(\Delta-2\Delta_1)\left\{2-\Gamma'[(Q_1^H+\Delta_1)(\Delta-\Delta_1)]-\Gamma'[(Q_2^H+\Delta-\Delta_1)\Delta_1]\right\}+$$
$$Q_1^H\left\{r\Gamma'[(Q_1^H+\Delta_1)(\Delta-\Delta_1)]+(1-r)\Gamma'[Q_1^H(\Delta-\Delta_1)]-r\right\}-$$
$$Q_2^H\left\{r\Gamma'[(Q_2^H+\Delta-\Delta_1)\Delta_1]+(1-r)\Gamma'[Q_2^H\Delta_1]-r\right\}. \tag{A.8}$$

When $Q_1^H=Q_2^H=Q^H/2$, (A.8) is zero when $\Delta_1=\Delta_2=\Delta/2$. Now instead suppose that $Q_1^H>Q_2^H$. One can easily show the derivative evaluated at $\Delta_1=\Delta/2$ is positive: the first line of (A.8) is zero, and the last two lines can be bounded below by a positive number in a manner similar to the argument above for the derivative in Q_1^H.

Moreover, one can also argue that the derivative is positive at $\Delta_1<\Delta/2$ when $Q_1^H>Q_2^H$. First, the first line of (A.8) is positive since $\Gamma'<1$. Second, note that $(Q_1^H+\Delta_1)(\Delta-\Delta_1)>(Q_2^H+\Delta-\Delta_1)\Delta_1$ whenever $\Delta_1<\frac{Q_1^H}{Q^H}\Delta$, which is greater than $\Delta/2$ by the assumption $Q_1^H>Q_2^H$. Also $Q_1^H(\Delta-\Delta_1)>Q_2^H\Delta_1$ by assumption. So by monotonicity of Γ', we obtain a positive derivative at $\Delta_1<\Delta/2$.

Thus the maximizers of $\pi(Q_1^H,\Delta_1)$ either satisfy $Q_1^{H*}=Q^H/2$, $\Delta_1^*=\Delta/2$ or $Q_1^{H*}>Q^H/2$, $\Delta_1^*>\Delta/2$. By the arguments above there exists a set $X=\{(Q_1^H,\Delta_1)\mid Q^H/2+\varepsilon\geq Q_1^H\geq Q^H/2, \Delta/2+\varepsilon\geq\Delta_1\geq\Delta/2\}\backslash\{(Q^H/2,\Delta/2)\}$ for some $\varepsilon>0$ such that the gradient of $\pi(Q_1^H,\Delta_1)$ is positive for all $x\in X$. Hence $Q_1^{H*}=Q^H/2$, $\Delta_1^*=\Delta/2$ cannot be a solution. ■

A.6 Proof of Proposition 4

Proof. We first analyze the program

$$\max_{\{Q_i(c_i),T_i(c_i)\}_{i=1,2;c_i\in\{0,c\}}}\sum_{i=1}^{2}rT_i(0)+(1-r)T_i(c)\ \text{ such that} \tag{A.9}$$

$$\min\{\pi_i(c_i,0,c_i),\pi_i(c_i,c,c_i)\}\geq 0 \tag{LL}$$

$$U\left[L_i(c_i\mid c_i)\right]\geq U\left[L_i(c_j\mid c_i)\right]\ \text{ for } c_j\neq c_i \tag{IC}$$

$$\overline{Q}\geq Q_i(c_i)\geq 0. \tag{QQ}$$

Following exactly the same arguments as in the proof of lemma 1, we can show that $\Delta_i \geq 0$ is necessary and sufficient for incentive compatibility, and that one can ignore the IC constraint for the high-cost firm. $\Delta_i \geq 0$ also implies one only needs to consider the LL constraints corresponding to meeting an efficient competitor. Clearly then it is optimal to choose

$$T_i(c) \ = \ Q_i^H[P(Q^H + \Delta_j) - c].$$

These observations allow us to re-write program (A.9) as

$$\max_{\{Q_i(c_i),T_i(c_i)\}_{i=1,2;c_i\in\{0,c\}}} \sum_{i=1}^{2} rT_i(0) + (1-r)T_i(c) \ \text{ such that} \tag{A.10}$$

$$Q_i^H[P(Q^H + \Delta_j) - c] - T_i(c) = 0 \tag{A.11}$$

$$(Q_i^H + \Delta_i)P(Q^H + \Delta) - T_i(0) \geq 0 \tag{A.12}$$

$$\begin{aligned} r(Q_i^H + \Delta_i)P(Q^H + \Delta) + (1-r)(Q_i^H + \Delta_i)P(Q^H + \Delta_i) - T_i(0) \geq \\ rQ_i^H P(Q^H + \Delta_j) + (1-r)Q_i^H P(Q^H) - T_i(c) \end{aligned} \tag{A.13}$$

$$\Delta_i \geq 0, Q_i^H \geq 0. \tag{A.14}$$

where (A.11) is the binding limited-liability constraint for the high-cost firm corresponding to meeting an efficient competitor; (A.12) is the limited-liability constraint for the low-cost firm; (A.13) is the incentive-compatibility constraint for the low-cost firm; and (A.14) are non-negativity constraints. Our strategy for solving this problem is to ignore (A.12) and (A.14), and to solve the resulting relaxed program. Clearly in this relaxed program the IC constraint for the low-cost firm will be binding since otherwise the upstream firm could increase profit by increasing $T_i(0)$. So we can write the simplified program as

$$\max_{\{Q_i(c_i),T_i(c_i)\}_{i=1,2;c_i\in\{0,c\}}} \sum_{i=1}^{2} rT_i(0) + (1-r)T_i(c) \ \text{ such that} \tag{A.15}$$

$$Q_i^H[P(Q^H + \Delta_j) - c] - T_i(c) = 0 \tag{A.16}$$

$$\begin{aligned} r(Q_i^H + \Delta_i)P(Q^H + \Delta) + (1-r)(Q_i^H + \Delta_i)P(Q^H + \Delta_i) - T_i(0) = \\ rQ_i^H P(Q^H + \Delta_j) + (1-r)Q_i^H P(Q^H) - T_i(c) \end{aligned} \tag{A.17}$$

The strategy for the rest of the proof is to derive conditions under which exclusive contracts solve (A.15). These contracts also solve (A.10) if they satisfy the limited-liability constraint for the low-cost firm (we will show below they satisfy the non-negativity constraints). We now argue that exclusive contracts indeed do so. Clearly this is the case for the low-cost firm that produces nothing. For the firm that produces in equilibrium, first note that the transfer for the high-cost firm is given by $T_i(c) = Q^H[P(Q^H) - c]$. When we plug this expression into the IC constraint for the low-cost firm, we obtain

$$T_i(0) = (Q^H + \Delta)[rP(Q^H + \Delta) + (1-r)P(Q^H + \Delta)] - Q^H P(Q^H) + Q^H P(Q^H) - cQ^H = (Q^H + \Delta)P(Q^H + \Delta) - cQ^H$$

which is less than $(Q^H + \Delta)P(Q^H + \Delta)$ since $Q^H \geq 0$. Hence exclusive contracts that solve (A.15) satisfy low-cost limited liability, and so are also solutions to (A.10).

We proceed with the solution to program (A.15). The upstream objective from plugging $T_i(c)$ and $T_i(0)$ into the objective function is

$$\begin{aligned} & r^2 \left(Q^H + \Delta\right) P\left(Q^H + \Delta\right) - r(1-r)Q^H P\left(Q^H\right) - cQ^H + \\ & r(1-r)\left[\left(Q_1^H + \Delta_1\right) P\left(Q^H + \Delta_1\right) + \left(Q_2^H + \Delta_2\right) P\left(Q^H + \Delta_2\right)\right] + \\ & (1-r^2)[Q_1^H P\left(Q^H + \Delta_2\right) + Q_2^H P\left(Q^H + \Delta_1\right)]. \end{aligned}$$

Without loss of generality, let $\Delta_1 \geq \Delta_2$, in which case $Q_1^H = Q^H$ is optimal (the solution is unique when $\Delta_1 > \Delta_2$). The upstream objective function can thus be written as

$$\begin{aligned} \pi(Q^H, \Delta, \Delta_2) = \ & r^2 R\left(Q^H + \Delta\right) - r(1-r)R\left(Q^H\right) - cQ^H + \\ & r(1-r)\left[R\left(Q^H + \Delta - \Delta_2\right) + R\left(Q^H + \Delta_2\right)\right] + (1-r)Q^H P\left(Q^H + \Delta_2\right) \end{aligned}$$

where $R(Q) \equiv QP(Q)$ is revenue. The solutions we study below will all satisfy $\Delta/2 \geq \Delta_2$. The partial derivatives of $\pi(Q^H, \Delta, \Delta_2)$ are

$$\frac{\partial \pi(Q^H, \Delta, \Delta_2)}{\partial Q^H} = \left\{ \begin{array}{c} r^2 \mathrm{MR}\left(Q^H + \Delta\right) - r(1-r)\mathrm{MR}\left(Q^H\right) + \\ r(1-r)\left[\mathrm{MR}\left(Q^H + \Delta - \Delta_2\right) + \mathrm{MR}\left(Q^H + \Delta_2\right)\right] + \\ (1-r)\left[P\left(Q^H + \Delta_2\right) + Q^H P'\left(Q^H + \Delta_2\right)\right] \end{array} \right\} - c \quad \text{(A.18)}$$

$$\frac{\partial \pi(Q^H, \Delta, \Delta_2)}{\partial \Delta} = r^2 \mathrm{MR}\left(Q^H + \Delta\right) + r(1-r)\mathrm{MR}\left(Q^H + \Delta - \Delta_2\right) \quad \text{(A.19)}$$

$$\frac{\partial \pi(Q^H, \Delta, \Delta_2)}{\partial \Delta_2} = \left\{ \begin{array}{c} r(1-r)\left[-\mathrm{MR}\left(Q^H + \Delta - \Delta_2\right) + \mathrm{MR}\left(Q^H + \Delta_2\right)\right] + \\ (1-r)Q^H P'\left(Q^H + \Delta_2\right) \end{array} \right\}. \quad \text{(A.20)}$$

Solution I: $Q^H = 0$, $\Delta_2 = \frac{\Delta}{2}$. This solution exists only if there is some $\Delta > 0$ such that the partial derivatives evaluated at $(Q^H, \Delta, \Delta_2) = (0, \Delta, \Delta/2)$ satisfy $\frac{\partial \pi(Q^H, \Delta, \Delta_2)}{\partial Q^H} < 0$, and $\frac{\partial \pi(Q^H, \Delta, \Delta_2)}{\partial \Delta} = \frac{\partial \pi(Q^H, \Delta, \Delta_2)}{\partial \Delta_2} = 0$. (A.20) is clearly satisfied, while (A.18) and (A.19) rewrite as[18]

$$-r(1-r)\mathrm{MR}(0) + r(1-r)\mathrm{MR}\left(\frac{\Delta}{2}\right) + (1-r)P\left(\frac{\Delta}{2}\right) < c. \quad \text{(A.21)}$$

$$r\mathrm{MR}(\Delta) + (1-r)\mathrm{MR}\left(\frac{\Delta}{2}\right) = 0 \quad \text{(A.22)}$$

This solution cannot exist for r sufficiently small since $\mathrm{MR}\left(\frac{\Delta}{2}\right) = 0$ and $P\left(\frac{\Delta}{2}\right) < c$ cannot hold

[18] Here we have also plugged (A.18) into (A.19).

simultaneously by assumption.

Solution II: $Q^H > 0$, $0 < \Delta_2 \leq \frac{\Delta}{2}$. This solutions exists only if the partial derivatives are all 0. The resulting system of equations simplifies to

$$\left\{ \begin{array}{c} -r(1-r)\text{MR}\left(Q^H\right) + r(1-r)\text{MR}\left(Q^H + \Delta_2\right) + \\ (1-r)\left[P\left(Q^H + \Delta_2\right) + Q^H P'\left(Q^H + \Delta_2\right)\right] \end{array} \right\} = c \quad \text{(A.23)}$$

$$r\text{MR}\left(Q^H + \Delta\right) + (1-r)\text{MR}\left(Q^H + \Delta - \Delta_2\right) = 0 \quad \text{(A.24)}$$

$$r(1-r)\left[-\text{MR}\left(Q^H + \Delta - \Delta_2\right) + \text{MR}\left(Q^H + \Delta_2\right)\right] + (1-r)Q^H P'\left(Q^H + \Delta_2\right) = 0. \quad \text{(A.25)}$$

Since $P' < 0$, (A.25) implies that $\text{MR}\left(Q^H + \Delta_2\right) > \text{MR}\left(Q^H + \Delta - \Delta_2\right)$ which in turn implies $\Delta_2 < \Delta - \Delta_2$ and $\Delta_2 < \frac{\Delta}{2}$. As r approaches 0, the left hand side of (A.25) must be strictly negative. So this solution cannot exist for r sufficiently small.

Solution III: Exclusive contract with $Q^H > 0$, $\Delta > 0$, $\Delta_2 = 0$. This solution exists only if

$$r\text{MR}\left(Q^H + \Delta\right) + (1-r)\text{MR}\left(Q^H\right) = c \quad \text{(A.26)}$$

$$r\text{MR}\left(Q^H + \Delta\right) = 0 \quad \text{(A.27)}$$

$$r(1-r)\left[-\text{MR}\left(Q^H + \Delta\right) + \text{MR}\left(Q^H\right)\right] + (1-r)Q^H P'\left(Q^H\right) < 0. \quad \text{(A.28)}$$

which simplifies to

$$(1-r)\text{MR}\left(Q^H\right) = c \quad \text{(A.29)}$$

$$\text{MR}\left(Q^H + \Delta\right) = 0 \quad \text{(A.30)}$$

$$r\text{MR}\left(Q^H\right) + Q^H P'\left(Q^H\right) < 0. \quad \text{(A.31)}$$

This solution clearly exists when r is small.

Note that we have not considered a solution in which $\Delta = 0$. To rule this out, one can follow the exact same logic as in the proof of proposition 1. ■

A.7 Proof of Proposition 5

Proof. Following the same steps as in Lemma 1, we can without loss of generality ignore the high-cost incentive compatibility constraint and replace the maximization problem in (11) with one in which $\Delta_i \geq 0$ and the incentive compatibility constraint of the low-cost firm is satisfied.

We begin with the case of risk neutrality. The ex-ante utility for firm i is

$$r\left[\left(Q_i^H + \Delta_i\right)\left\{[r + (1-r)\rho]P\left(Q^H + \Delta\right) + (1-r)(1-\rho)P\left(Q^H + \Delta_i\right)\right\} - T_i(0)\right] +$$
$$(1-r)\left[Q_i^H\left\{r(1-\rho)P\left(Q^H + \Delta_j\right) + [(1-r) + r\rho]P\left(Q^H\right) - c\right\} - T_i(c)\right] = 0 \quad \text{(A.32)}$$

from which we get

$$\sum_{i=1}^{2} rT_i(0) + (1-r)T_i(c) =$$
$$r[r+(1-r)\rho]\left(Q^H+\Delta\right)P\left(Q^H+\Delta\right) + (1-r)[(1-r)+r\rho]Q^H P\left(Q^H\right) - (1-r)cQ^H +$$
$$r(1-r)(1-\rho)\left[\left(Q^H+\Delta_1\right)P\left(Q^H+\Delta_1\right) + \left(Q^H+\Delta-\Delta_1\right)P\left(Q^H+\Delta-\Delta_1\right)\right].$$

One can implement any interim allocation with $\Delta_i \geq 0$ using transfers satisfying

$$\begin{aligned} T_i(0) - T_i(c) \ = \ & \left(Q_i^H+\Delta_i\right)\left\{[r+(1-r)\rho]P\left(Q^H+\Delta\right) + (1-r)(1-\rho)P\left(Q^H+\Delta_i\right)\right\} - \\ & Q_i^H\left\{[r+(1-r)\rho]P\left(Q^H+\Delta_j\right) + (1-r)(1-\rho)P\left(Q^H\right)\right\} \end{aligned}$$

and (A.32). Showing the optimal values of Δ_1 and Δ_2 can be done using arguments nearly identical to those in the proof of proposition 1.

We now turn to the case of infinite risk aversion. The first step is to argue which outcome must be worst *ex ante* for the firm. Since incentive compatibility requires $\Delta_i \geq 0$, meeting a high cost firm can never be strictly better than meeting a low cost firm. The payoff from meeting a low cost firm when low cost is $\left(Q^H+\Delta_i\right)P\left(Q^H+\Delta\right) - T_i(0)$ and from meeting a low cost firm when high cost is $Q^H P\left(Q^H+\Delta_j\right) - cQ^H - T_i(c)$. Now, interim incentive compatibility requires that

$$\left(Q^H+\Delta_i\right)P\left(Q^H+\Delta\right) - T_i(0) \geq Q^H P\left(Q^H+\Delta_j\right) - T_i(c) > Q^H P\left(Q^H+\Delta_j\right) - cQ^H - T_i(c).$$

So incentive compatibility implies drawing the high cost and meeting a low cost is the worst possible outcome *ex ante*. Thus the ex ante participation constraint is

$$T_i(c) \ = \ Q_i^H P\left[Q^H+\Delta_j\right] - cQ_i^H \tag{A.33}$$

which gives the optimal value of $T_i(c)$. The constraint on $T_i(0)$ is thus incentive compatibility, yielding

$$T_i(0) \ = \ \left(Q_i^H+\Delta_i\right)P\left[Q^H+\Delta\right] - cQ_i^H. \tag{A.34}$$

Plugging into the upstream objective function gives

$$\begin{aligned} & r\left[\left(Q_1^H+\Delta_1\right)P\left(Q^H+\Delta\right) + \left(Q_2^H+\Delta_2\right)P\left(Q^H+\Delta\right)\right] - cQ^H + \\ & (1-r)\left[Q_1^H P\left(Q^H+\Delta_2\right) + Q_2^H P\left(Q^H+\Delta_1\right)\right] \\ = \ & r\left(Q^H+\Delta\right)P\left(Q^H+\Delta\right) - cQ^H + \\ & (1-r)\left[Q_1^H P\left(Q^H+\Delta_2\right) + Q_2^H P\left(Q^H+\Delta_1\right)\right]. \end{aligned} \tag{A.35}$$

This is precisely the objective function analyzed in the case of infinite risk aversion in Section 3.1 (see expression 9). ■

A.8 Proof of Proposition 6

Proof. We begin by deriving the optimal outputs under limited liability when the upstream firm offers the optimal exclusive contract to a single firm. From the proof of proposition 4, the equations that define the optimal Q^H and Δ in an exclusive contract are $MR(Q^{H*}+\Delta^*) = 0$ and $(1-r)MR(Q^{H*}) = c$. With linear demand, $MR(Q) = 1-2Q$. Solving gives $Q^{H*} = \frac{1-c}{2} - \frac{r}{1-r}\frac{c}{2}$ and $\Delta^* = \frac{c}{2(1-r)}$. By definition $Q^{LL}(c) = Q^{H*}$ and $Q^{LL}(0) = Q^{H*} + \Delta^*$.

To derive the optimal outputs with revenue sharing, we first elaborate on the claim in the main text that we can express the optimal transfers as $T_i(c) = T_i(0) = -cQ_i(c)$. First note that we can drop the participation constraint of the low-cost firm since

$$-T_i(0) \geq -T_i(c) \geq -cQ_i(c) - T_i(c) \geq 0,$$

where the first inequality comes from the low-cost IC constraint and the final inequality comes from the high-cost PC constraint. As in the proof of lemma 1, consider the relaxed program with only the high-cost PC and low-cost IC constraints:

$$-cQ_i(c) - T_i(c) \geq 0 \tag{A.36}$$

$$-T_i(0) \geq -T_i(c) \tag{A.37}$$

Clearly in an optimal contract both these are binding, since otherwise the upstream firm could increase transfers and thus profit. From the high-cost PC constraint we obtain $T_i(c) = -cQ_i(c)$, and from the low-cost IC constraint we obtain $T_i(0) = T_i(c)$. Moreover, the ignored IC constraint for the high-cost firm writes as

$$c[Q_i(0) - Q_i(c)] \geq T_i(c) - T_i(0).$$

The right-hand side of this expression is zero in the optimal contract in the relaxed program, and so the solution to the relaxed program is also the solution to the full program whenever $Q_i(0) \geq Q_i(c)$, which is true in the solution as shown below.

After plugging $T_i(c) = T_i(0) = -cQ_i(c)$ into (12), we obtain exactly expression (6). As argued in section 2.2, we can restrict attention to $\Delta_1 = \Delta_2 = \frac{\Delta}{2}$, and solve for the optimal Q^H and Δ. The respective first order conditions are

$$r^2 MR(Q^{H*} + \Delta^*) + 2r(1-r)MR(Q^{H*} + \Delta^*/2) + (1-r)^2 MR(Q^{H*}) = c \tag{A.38}$$

$$r^2 MR(Q^{H*} + \Delta^*) + r(1-r)MR(Q^{H*} + \Delta^*/2) = 0 \tag{A.39}$$

which rewrite with linear demand as $1 - 2Q^{H*} - 2r\Delta^* = c$ and $r[1 - 2Q^{H*}] - (r^2 + r)\Delta^* = 0$. This system has solution

$$Q^{H*} = \frac{1-c-r-rc}{2(1-r)}, \; \Delta^* = \frac{c}{1-r}.$$

By definition $Q^{RS}(c) = Q^{H*}/2$ and $Q^{RS}(0) = (Q^{H*} + \Delta^*)/2$.

We need to assume that the parameters are such that the upstream firm wishes to contract a positive Q^{H*}. The condition that satisfies expressions (A.29)-(A.31) in the proof of proposition 4 in the case of linear demand is $r < \frac{1-c}{1+2c}$, while the condition that guarantees that $Q^{RS}(c) > 0$ is $r < \frac{1-c}{1+c}$. So we assume that $r < \frac{1-c}{1+2c}$ in the computations below, which can be guaranteed for any $r \in (0,1)$ for small enough c.

If we let Q be aggregate output, then upstream profit is $\mathbb{E}[\,Q\,] - \mathbb{E}[\,Q\,]^2 - V[\,Q\,] - c\mathbb{E}[\,Q\,]$. Moreover, as shown in the text in section 2.2, $V[\,Q\,] = r(1-r)\sum_i \Delta_i^2$. The variance of aggregate output with the optimal exclusive contract minus the optimal revenue sharing contract is

$$r(1-r)\left[\frac{1}{2} - \frac{1-c}{2} + \frac{r}{1-r}\frac{c}{2}\right]^2 - 2r(1-r)\left[\frac{1}{4} + \frac{c}{4} - \frac{1-c}{4} + \frac{r}{1-r}\frac{c}{2}\right]^2 =$$
$$-r(1-r)\left[\left(\frac{r}{1-r} + 1\right)\frac{c}{2}\right]^2 = -\frac{r}{1-r}\frac{c^2}{4}.$$

The difference in expected production costs is

$$c\left(\frac{1-c}{2} - \frac{r}{1-r}\frac{c}{2}\right) - 2c\left(\frac{1-c}{4} - \frac{r}{1-r}\frac{c}{2}\right) =$$
$$c\left(\frac{1-c}{2} - \frac{r}{1-r}\frac{c}{2}\right) - c\left(\frac{1-c}{2} - \frac{r}{1-r}c\right) = \frac{r}{1-r}\frac{c^2}{2}.$$

So overall the upstream firm is better off by $\frac{r}{1-r}\frac{c^2}{4}$ with revenue sharing.

Expected downstream profits are given by $rc\mathbb{E}[\,Q\,]$. So the difference between exclusion and revenue sharing is

$$rc\left(\frac{1-c}{2} - \frac{r}{1-r}\frac{c}{2}\right) - 2cr\left(\frac{1-c}{4} - \frac{r}{1-r}\frac{c}{2}\right) =$$
$$rc\left(\frac{1-c}{2} - \frac{r}{1-r}\frac{c}{2}\right) - rc\left(\frac{1-c}{2} - \frac{r}{1-r}c\right) = \frac{r}{1-r}\frac{rc^2}{2}.$$

Expected consumer surplus is $\frac{\mathbb{E}[\,Q^2\,]}{2} = \frac{\mathbb{E}[\,Q\,]^2 + V[\,Q\,]}{2}$. So the difference between exclusion and revenue sharing is

$$\frac{1}{2}\left[r(1-r)\left[\frac{1}{2} - \frac{1-c}{2} + \frac{r}{1-r}\frac{c}{2}\right]^2 - 2r(1-r)\left[\frac{1}{4} + \frac{c}{4} - \frac{1-c}{4} + \frac{r}{1-r}\frac{c}{2}\right]^2\right]$$
$$= -\frac{r}{1-r}\frac{c^2}{8}$$

The welfare comparisons are straightforward given these expressions. ■

A.9 Proof of Proposition 7

Proof. We know that the optimal exclusionary solution can be implemented if IC_{LH} and IC_{LL} are satisfied. First, consider IC_{LH}. It can be rewritten as $P(Q^{H*}) - P\left(2Q^{H*} + \Delta^*\right) \geq c$. The LHS is always strictly positive because of decreasing demand. (Note that Q^{H*} and Δ^* in general depend on c, but as $c \to 0$, Q^{H*} will always be strictly positive whereas $\lim_{c\to 0} \Delta^* = 0$. This follows immediately by inspection of the FOCs implicitly defining Q^{H*} and Δ^*.) Define $\inf \left[P(Q^{H*}) - P\left(2Q^{H*} + \Delta^*\right) \right]$ as the lowest value that the LHS can attain, and set $\bar{c}_1 = \inf \left[P(Q^{H*}) - P\left(2Q^{H*} + \Delta^*\right) \right]$. For any $c \leq \bar{c}_1$, the IC_{LH} is satisfied.

Next, consider IC_{LL}. It can be rewritten as $\left[P(Q^{H*} + \Delta^*) - P\left(2Q^{H*} + 2\Delta^*\right) \right] (Q^{H*} + \Delta^*)/Q^{H*} \geq c$. Since $(Q^{H*} + \Delta^*)/Q^{H*} \geq 1$, if $\left[P(Q^{H*} + \Delta^*) - P\left(2Q^{H*} + 2\Delta^*\right) \right] \geq c$, the IC_{LL} will be satisfied. The LHS of the last inequality is always strictly positive because of decreasing demand. Define $\inf \left[P(Q^{H*} + \Delta^*) - P\left(2Q^{H*} + 2\Delta^*\right) \right]$ as the lowest value that the LHS can attain, and set $\bar{c}_2 = \inf \left[P(Q^{H*} + \Delta^*) - P\left(2Q^{H*} + 2\Delta^*\right) \right]$. For any $c \leq \bar{c}_2$, the IC_{LL} is satisfied.

Define $\bar{c} = \min \{\bar{c}_1, \bar{c}_2\}$. If $c \leq \bar{c}$, both ICs are satisfied: the principal is able to implement the exclusionary solution, with only one firm selling in equilibrium, by making use of uniform contracts. ■